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Analytical Models for Velocity Distributions in Compound Channels with Emerged and Submerged Vegetated Floodplains

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Abstract: The lateral distributions of depth-averaged velocity in open compound channels with emerged and submerged vegetated floodplains were analyzed based on the analytical solution of the depth-integrated Reynolds-Averaged Navier-Stokes equation with a term to account for the effects of vegetation. The three cases considered for open channels were two-stage rectangular channel with emerged vegetated floodplain, rectangular channel with submerged vegetated corner, and two-stage rectangular channel with submerged vegetated floodplain, respectively. To predict the depth-averaged velocity with submerged vegetated floodplains, we proposed a new method based on a two-layer approach where flow above and through the vegetation layer was described separately. Moreover, further experiments in the two-stage rectangular channel with submerged vegetated floodplain were carried out to verify the results. The analytical solutions of the cases indicated that the corresponding analytical depth-averaged velocity distributions agree well with the simulated and experimental prediction. The analytical solutions of the cases with theoretical foundation and without programming calculation were reasonable and applicable, which were more convenient than numerical simulations. The analytical solutions provided a way for future researches to solve the problems of submerged vegetation and discontinuous phenomenon of depth-averaged velocity at the stage point for compound channels. Understanding the hydraulics of flow in compound channels with vegetated floodplains is very important for supporting the management of fluvial processes.

Keywords: compound channel; velocity distribution; vegetated floodplain; two-stage rectangular channel; analytical solution

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1 Introduction

Natural rivers are commonly characterized by a main channel for primary flow conveyance and a floodplain, often partially covered with vegetation such as shrubs or trees, to carry extra flow during floods (Tang and Knight, 2009). It is essential for the practical engineer to determine velocity distributions and thus discharge capacity for floodplain management, river training works or water resources activities that account for such processes (Fischer-Antze *et al*., 2001). Besides, understanding the hydraulics of flow in a compound channel with vegetated floodplains is very important for determining the stage-discharge curve and for supporting the man-

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agement of fluvial processes (Yang *et al.*, 2007).

Up to now, various physical laboratory models have been proposed for compound channels with and without vegetation on the floodplain, and a lot of analytical and numerical models have also been developed (Knight and Hamed, 1984; Pasche, 1985; Shiono and Feng, 2003; Yang *et al.*, 2007; Tanino and Nepf, 2008; White and Nepf, 2008; Sun and Shiono, 2009; Sun *et al.*, 2013; Tang *et al.*, 2014). The hydrodynamic response of turbulent flow in a compound wide rectangular open channel with a vegetated domain at the channel bank, in a channel corner and a floodplain, to the vegetation density and diameter was discussed (Naot *et al.*, 1996). To this end a phenomenological model was embedded in an algebraic stress model with the vegetation modeled as an internal resistance that exerts drag force, produced energy of turbulence, and interfered with its anisotropy and length scale. The practical method to predict depth-averaged velocity and shear stress for straight and meandering overbank flows was presented (Ervine *et al.*, 2000). Velocity distributions in channels partially covered with vegetation were computed (Fischer-Antze *et al.*, 2001) with a three-dimensional model. And based on the semi-implicit method for pressure-linked equations (SIMPLE) and the k-ε turbulence model, the Navier-Stokes equations were solved. The vegetation was modeled as vertical cylinders. A formula for the drag force on the vegetation was included as a sink term in the Navier-Stokes equation. The mechanism of the dip phenomenon whereby the location of the maximum velocity appeared below the free surface towards the secondary currents in open-channel flows was investigated (Yang *et al.*, 2004). A Reynolds stress model for the numerical simulation of compound open-channel flows with vegetation on the floodplain was described (Kang and Choi, 2006). These data of physical laboratory models and numerical simulations provide an important reference to further researches. Nevertheless, physical laboratory models for two-stage rectangular channels with submerged vegetated floodplains are few. In this study, an experiment was carried out for the two-stage rectangular channel with submerged vegetated floodplains.

There are many studies on vertical distributions of velocity in case of submerged vegetation. Analytical solution of the flow velocity in case of submerged rigid cylindrical vegetation was proposed (Huthoff *et al.*, 2007). The model was based on a two-layer approach where flow above and through the vegetation layer was described separately and the three types of prismatic geometry investigated were: simple rectangular, symmetric, and asymmetric rectangular compound channels. Based on the detailed laboratory experiments and theoretical analysis, a new three-layer model was proposed (Huai *et al.*, 2009a) to predict the vertical velocity distribution in an open channel flow with submerged vegetation. By summarizing previous studies on the flow in open channels with rigid vegetation, a mathematical model for submerged and emerged rigid vegetation was constructed (Huai *et al.*, 2009b). A series of experiments in an open channel with fully submerged shrubs and found that non-uniformity of vegetation diameter leads to non-uniform vertical velocity profiles within vegetation were carried out (Hui *et al.*, 2009). An analytical solution for the vertical profiles of the horizontal velocity of channel flow with submerged shrub-like vegetation was investigated (Liu *et al.*, 2012). These studies on vertical distributions of velocity are very important to determine the relation between depth-averaged flow velocities above and through the vegetation layer for submerged vegetation. Nevertheless, there are few researches applying the relation of depth-averaged flow velocities in compound channels with submerged vegetated floodplains. In this study, based on the relation of depth-averaged flow velocities, an analytical model for lateral distributions of depth-averaged velocity in compound channels with submerged vegetated floodplains was successfully established.

There are also many analytical and numerical models for lateral distributions of depth-mean velocity in case of compound channel with vegetated floodplain. Lateral distributions of depth-mean velocity and boundary shear stress were derived theoretically for channels of any shape, provided that the boundary geometry can be separated into linear elements (Shiono and Knight, 1991). The analytical model included the effects of bed-generated turbulence, lateral shear turbulence and secondary flows. The quasi two-dimensional (2D) model that calculates depth-averaged velocity and bed shear stress in a straight compound channel with a vegetated floodplain was presented (Rameshwaran and Shiono, 2007). Three analytical stage-discharge formulas, suitable for hand calculation, were derived for prismatic open channels (Liao and Knight, 2007). Later, the two-dimensional analytical solutions for compound channel flows with vegetated floodplains were proposed (Huai *et al*., 2008; Huai *et al*., 2009c). Tang and Knight (2009) and Sterling *et al*. (2011) proposed a method for predicting the depth-averaged velocity in compound channels with emerged vegetated floodplains based on an analytical solution of the depth-integrated Reynolds-Averaged Navier-Stokes equation with a term included to account for the effects of vegetation. The vegetation was modeled via an additional term in the momentum equation to account for the additional drag force.

There have been several studies on lateral distributions of depth-averaged velocity in rectangular channels or compound channels with emerged vegetated floodplain. Nevertheless, due to the complexity of submerged vegetation, there are few researches on the lateral distribution of depth-averaged velocity with submerged vegetated floodplain. Therefore, we focused on the lateral distribution of depth-averaged velocity in the main channel without vegetation and floodplain with submerged vegetation. The analytical solutions of lateral distributions for three types of compound open channel were proposed first. The compound open channels were Case 1: asymmetric two-stage rectangular channel with emerged vegetated floodplain; Case 2: rectangular channel with submerged vegetated corner; Case 3: symmetric two-stage rectangular channel with submerged vegetated floodplain, respectively. Subsequently, a corresponding experiment was also carried out for the two-stage rectangular channel with submerged vegetated floodplains.

2 Theoretical Analysis and Experimental Data

2.1 Theoretical analysis for lateral distribution of depth-averaged velocity

Our research focused on the lateral distribution of depth-averaged velocity in the main channel without vegetation and floodplain with submerged vegetation as shown in Fig. 1.

Due to the effects of vegetation, an additional momentum term (the drag force term, F_v) was introduced into the Reynolds-Averaged Navier-Stokes (RANS) equation, which when combined with the continuity equation gave the following equation:

$$\rho[\frac{\partial(U^2)}{\partial x}+\frac{\partial(UV)}{\partial y}+\frac{\partial(UW)}{\partial z}]=\rho g S_0+\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}-F_v \tag{1}$$

where $\{U, V, W\}$ are the velocity components in the $\{x, y, z\}$ directions, x is the streamwise coordinate parallel to the channel bed, y is the lateral coordinate and z is the coordinate normal to the bed, $\{\tau_{xx}, \tau_{yx}, \tau_{zx}\}$ are the Reynolds stresses on planes perpendicular to the x, y and z directions, respectively, ρ is the fluid density, g is the acceleration due to gravity, and S_0 is the bed slope of the channel. F_v is the drag force per unit fluid volume due to the vegetation, and it is represented as

$$F_v=\frac{1}{2}\rho(C_D\beta A_v)U^2 \tag{2}$$

where C_D is the drag coefficient, β is a shape factor to

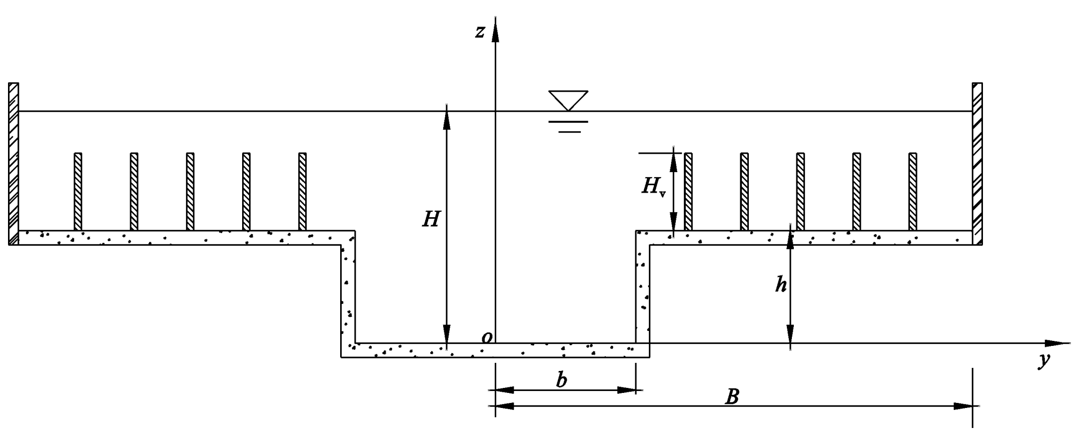

Fig. 1 Schematic description of open channel flow. H is water depth, H_v is vegetation height, h is bankfull height, y is the lateral coordinate, z is the coordinate normal to the bed, B is half of width of the channel, b is half of main channel width

account for the type of vegetation and A_v is the projected area of the vegetation in the streamwise direction per unit volume.

Integrating equations (1) and (2) over the water depth (H), assuming $W = 0$ at $z = 0$ and $z = H$ (Shiono and Knight, 1991) and also assuming that $\partial(HU_d^2)/\partial x \approx 0$ and $\partial(H\tau_{xx})/\partial x \approx 0$, gives

$$\rho\frac{\partial H(U_v)_d}{\partial y} = \rho gHS_0 + \frac{\partial H\overline{\tau}_{yx}}{\partial y} - \tau_b - \int_0^H \frac{1}{2}\rho(C_D\beta A_v)U^2\mathrm{d}z \tag{3}$$

$$\begin{aligned}\int_0^H \frac{1}{2}\rho(C_D\beta A_v)U^2\mathrm{d}z &= \int_0^{H_v} \frac{1}{2}\rho(C_D\beta A_v)U^2\mathrm{d}z + \\ \int_{H_v}^H \frac{1}{2}\rho(C_D\beta A_v)U^2\mathrm{d}z &= \int_0^{H_v} \frac{1}{2}\rho(C_D\beta A_v)U^2\mathrm{d}z \\ &= \frac{1}{2}\rho(C_D\beta A_v)H_vU_v^2\end{aligned} \tag{4}$$

in which, d refers to a depth-averaged value, $\overline{\tau}_{yx}$ is the depth-averaged Reynolds stresses on planes perpendicular to the y directions, τ_b is the bed shear stress, H_v is the vegetation height, U_v is the depth-averaged flow velocity in the vegetation layer, U_d is the depth-averaged flow velocity.

Rectangular channel with submerged rigid cylindrical vegetation is shown in Fig. 2a; it is different from rectangular channel with submerged vegetated corner. Analytical solution for the depth-averaged flow velocity in case of submerged rigid cylindrical vegetation is proposed (Huthoff *et al*., 2007). The lateral distribution of depth-averaged velocity for that case is invariable. Based on the relation between the depth-averaged flow velocity U_r and U_T (Huthoff *et al*., 2007), this paper assumes that U_v and U_d on the floodplain share a similar relation. The relations of depth-averaged velocity (Huthoff *et al*., 2007) are given as

$$\frac{U_r}{U_T} = \frac{\sqrt{H/H_v}}{\sqrt{\frac{H_v}{H} + \frac{H-H_v}{H}\left(\frac{H-H_v}{l}\right)^{\frac{2}{3}\left(1-\left(\frac{H}{H_v}\right)^{-5}\right)}}} \tag{5}$$

where U_r is depth-averaged flow velocity in the vegetation layer for submerged vegetation, U_T is depth- averaged flow velocity over total flow depth, l is the rod spacing as shown in Figs. 2a and 2b.

Assuming that the relation of U_v and U_d on the floodplain, R_u, is similar to the relation of Equation (5), it can be concluded that:

$$U_v = R_uU_d \tag{6}$$

$$R_u \approx \frac{\sqrt{H/H_v}}{\sqrt{\frac{H_v}{H} + \frac{H-H_v}{H}\left(\frac{H-H_v}{l}\right)^{\frac{2}{3}\left(1-\left(\frac{H}{H_v}\right)^{-5}\right)}}} \tag{7}$$

By considering Equation (6) with Equation (7), the last term of Equation (4) can be expressed as

$$\frac{1}{2}\rho(C_D\beta A_v)H_vU_v^2 = \frac{1}{2}\rho(C_D\beta A_v)H_vR_u^2U_d^2 \tag{8}$$

Substituting Equation (8) into Equation (3) gives

$$\rho\frac{\partial H(UV)_d}{\partial y} = \rho gHS_0 + \frac{\partial H\overline{\tau}_{yx}}{\partial y} - \tau_b - \frac{1}{2}\rho(C_D\beta A_v)H_vR_u^2U_d^2 \tag{9}$$

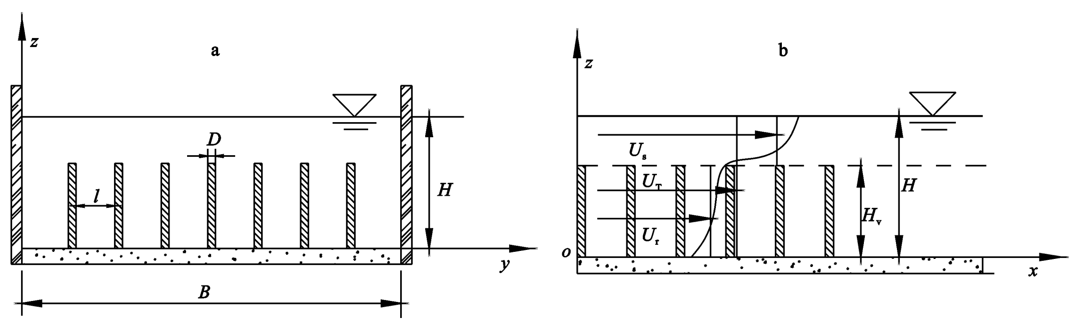

Fig. 2 Rectangular channel with submerged rigid cylindrical vegetation. (a) Cross-section profile, in which, y is the lateral coordinate, z is the coordinate normal to the bed, B is width of the channel, H is water depth, l is rod spacing. (b) Longitudinal profile, in which, x is the streamwise coordinate parallel to the channel bed, H_v is vegetation height, U_r is depth-averaged flow velocity in the vegetation layer for submerged vegetation, U_s is depth-averaged flow velocity in the surface layer, U_T is depth-averaged flow velocity over total flow depth

The blockage effect of the vegetation is taken into account via a porosity term, δ, which is related to the volumetric vegetation density, φ, via $\delta = 1 - \varphi$. φ is defined as the ratio of the volume of vegetation to the volume of the flow. It is beneficial to write Equation (9) in terms of an effective water volume, as

$$\rho\delta\frac{\partial H(UV)_{\mathrm{d}}}{\partial y} = \rho\delta g H S_0 + \delta\frac{\partial H\bar{\tau}_{yx}}{\partial y} - \delta\tau_{\mathrm{b}} - \frac{1}{2}\rho(C_{\mathrm{D}}\beta A_{\mathrm{v}})H_{\mathrm{v}}R_{\mathrm{u}}^2 U_{\mathrm{d}}^2 \tag{10}$$

Making the following assumptions that were originally derived by other researchers (Shiono and Knight, 1991; Tang and Knight, 2009):

$$\tau_{\mathrm{b}} = (\frac{f}{8})\rho U_{\mathrm{d}}^2 \tag{11}$$

$$\bar{\tau}_{yx} = \rho\bar{\varepsilon}_{yx}\frac{\partial U_{\mathrm{d}}}{\partial y} \tag{12}$$

$$\bar{\varepsilon}_{yx} = \lambda U^* H \tag{13}$$

$$\Gamma = \frac{\partial}{\partial y}[H(\rho UV)_{\mathrm{d}}] \tag{14}$$

where f is the bed friction factor, λ is the lateral eddy viscosity, U^* is the local shear velocity, Γ is the depth-averaged secondary flow term.

Equation (10) can be rearranged as

$$\rho g H S_0 - \rho\frac{f}{8}U_{\mathrm{d}}^2 - \frac{1}{2\delta}\rho(C_{\mathrm{D}}\beta A_{\mathrm{v}})H_{\mathrm{v}}R_{\mathrm{u}}^2 U_{\mathrm{d}}^2 + \frac{\partial}{\partial y}\{\rho\lambda H^2(\frac{f}{8})^{1/2}U_{\mathrm{d}}\frac{\partial U_{\mathrm{d}}}{\partial y}\} = \Gamma \tag{15}$$

Equation (15) can be solved analytically, provided that the appropriate boundary conditions are specified. The equation considers the submerged vegetation effect that was derived by the present study.

If drag coefficient C_{D}, the density of vegetation (φ), local friction factor (f), eddy viscosity (λ) and secondary flow term (Γ) are known, then the analytical solution of Equation (15) for U_{d} can be given as:

$$U_{\mathrm{d}} = [A_1 \mathrm{e}^{\gamma y} + A_2 \mathrm{e}^{-\gamma y} + k]^{1/2} \tag{16}$$

where A_1 and A_2 are coefficients and

$$k = \frac{gS_0 H - \Gamma/\rho}{\frac{f}{8} + \frac{1}{2\delta}(C_{\mathrm{D}}\beta A_{\mathrm{v}})H_{\mathrm{v}}R_{\mathrm{u}}^2} \tag{17}$$

$$\gamma = \sqrt{\frac{2}{\lambda}}(\frac{8}{f})^{1/4}\frac{1}{H}\sqrt{\frac{f}{8} + \frac{1}{2\delta}(C_{\mathrm{D}}\beta A_{\mathrm{v}})H_{\mathrm{v}}R_{\mathrm{u}}^2} \tag{18}$$

2.1.1 Case 1: asymmetric two-stage rectangular channel with emerged vegetated floodplain

As to compound open channel with side wall, the equation and analytical solution for lateral distribution of depth-averaged velocity is given (Tang and Knight, 2009). In the domain of side wall, the processing method is coordinate transformation. ξ is the local depth given by $\xi = H - (y - b) / s$ (for $y > 0$) and $\xi = H + (y + b) / s$ (for $y < 0$), where b is the main channel width, s is channel side slope of the bank.

However, for the two-stage rectangular compound open channel, which is different from the compound open channel with side wall, there is a discontinuous phenomenon of depth-averaged velocity at the stage point. We applied the method (Liao and Knight, 2007) for non-vegetated floodplain to the lateral distribution of two-stage rectangular channel with emerged vegetated floodplain as shown in Fig. 3.

With each section having a different constant water depth, H, and parameters f, λ and Γ, the equations for the main channel and floodplain can be respectively given as:

$$U_{\mathrm{d}}^{(1)} = [A_1 \mathrm{e}^{\gamma_1 y} + A_2 \mathrm{e}^{-\gamma_1 y} + k_1]^{1/2} \tag{19}$$

$$U_{\mathrm{d}}^{(2)} = [A_3 \mathrm{e}^{\gamma_2 y} + A_4 \mathrm{e}^{-\gamma_2 y} + k_2]^{1/2} \tag{20}$$

where the superscripts (1), (2) indicate the main channel and floodplain, respectively, A_1, A_2 A_3 and A_4 are coefficients and

$$k_1 = \frac{8gS_0 H}{f_1}(1 - \frac{\Gamma_1}{\rho g S_0 H}) \tag{21}$$

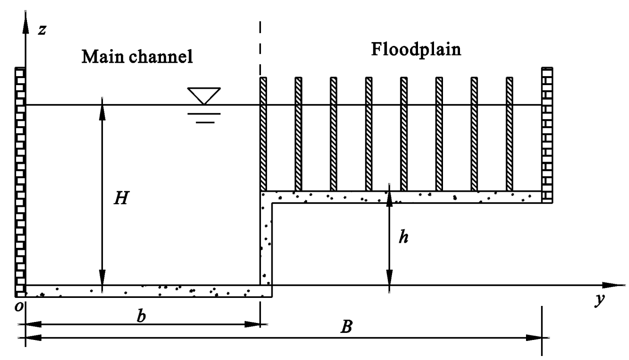

Fig. 3 Asymmetric two-stage rectangular channel with emerged vegetated floodplain. y is the lateral coordinate, z is the coordinate normal to the bed, b is main channel width, B is width of the channel, H is water depth, h is bankfull height

$$\gamma_1 = \sqrt{\frac{2}{\lambda_1}}(\frac{f_1}{8})^{1/4}\frac{1}{H} \tag{22}$$

$$k_2 = \frac{gS_0(H-h) - \Gamma_2/\rho}{\frac{f_2}{8} + \frac{C_D\beta A_v}{2\delta}(H-h)} \tag{23}$$

$$\gamma_2 = \sqrt{\frac{2}{\lambda_2}}(\frac{8}{f_2})^{1/4}\frac{\sqrt{\frac{f_2}{8} + \frac{1}{2\delta}(C_D\beta A_v)(H-h)}}{H-h} \tag{24}$$

Then the following boundary conditions were applied:

$$U_d^{(1)}\Big|_{y=0} = 0 \tag{25}$$

$$U_d^{(2)}\Big|_{y=B} = 0 \tag{26}$$

The boundary condition at $y = b$ is true for cases that are continuous in depth, but is not applicable here since the velocity of the main channel at this particular point is not continuous (Knight *et al*., 2004). However, to make the boundary condition similar to those cases where U_d is continuous, the condition can be written approximately as (Liao and Knight, 2007)

$$\alpha_u U_d^{(1)}\Big|_{y=b} = U_d^{(2)}\Big|_{y=b} \tag{27}$$

$$\frac{\partial U_d^{(1)}}{\partial y}\Bigg|_{y=b} = \frac{\partial U_d^{(2)}}{\partial y}\Bigg|_{y=b} \tag{28}$$

where α_u was presented by Liao and Knight (2007).

Applying these four boundary conditions to Equations (19) and (20), the linear equations can be derived as:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & e^{\gamma_2 B} & e^{-\gamma_2 B} \\ \alpha_u^2 e^{\gamma_1 b} & \alpha_u^2 e^{-\gamma_1 b} & -e^{\gamma_2 b} & -e^{-\gamma_2 b} \\ \alpha_u\gamma_1 e^{\gamma_1 b} & -\alpha_u\gamma_1 e^{-\gamma_1 b} & -\gamma_2 e^{\gamma_2 b} & \gamma_2 e^{-\gamma_2 b} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} -k_1 \\ -k_2 \\ -\alpha_u^2 k_1 + k_2 \\ 0 \end{bmatrix} \tag{29}$$

in which, coefficients A_1, A_2, A_3 and A_4 can be obtained by solving the system of linear equations. Substituting A_1, A_2, A_3 and A_4 into Equations (19) and (20) gave the lateral distribution of depth-average velocity in asymmetric two-stage rectangular channel with emerged vegetated floodplain.

2.1.2 Case 2: *rectangular channel with submerged vegetated corner*

Analytical solution for the flow velocity in case of submerged rigid cylindrical vegetation was proposed (Huthoff *et al*., 2007). In the paper, the lateral distribution of depth-averaged velocity in the main channel without vegetation and floodplain with submerged vegetation was considered.

For the rectangular channel with submerged vegetated corner shown in Fig. 4, the equations for the non-vegetated section and vegetated section can be given, respectively, which are the same as Equations (19) and (20), but the superscripts (1), (2) indicate non-vegetated section and vegetated section, respectively

$$k_1 = \frac{8gS_0H}{f_1}(1 - \frac{\Gamma_1}{\rho g S_0 H}) \tag{30}$$

$$\gamma_1 = \sqrt{\frac{2}{\lambda_1}}(\frac{f_1}{8})^{1/4}\frac{1}{H} \tag{31}$$

$$k_2 = \frac{gS_0H - \Gamma_2/\rho}{\frac{f_2}{8} + \frac{1}{2\delta}(C_D\beta A_v)H_v R_u^2} \tag{32}$$

$$\gamma_2 = \sqrt{\frac{2}{\lambda_2}}(\frac{8}{f_2})^{1/4}\frac{1}{H}\sqrt{\frac{f_2}{8} + \frac{1}{2\delta}(C_D\beta A_v)H_v R_u^2} \tag{33}$$

The appropriate boundary conditions were: (1) $U_d^{(1)} = 0$ at $y = 0$, (2) $U_d^{(2)} = 0$ at $y = B$, (3) $U_d^{(1)} = U_d^{(2)}$ at $y = b$, (4) $\partial U_d^{(1)}/\partial y = \partial U_d^{(2)}/\partial y$ at $y = b$. Applying these four boundary conditions to Equations (19) and (20), the coefficients A_1, A_2, A_3 and A_4 can be obtained by solving

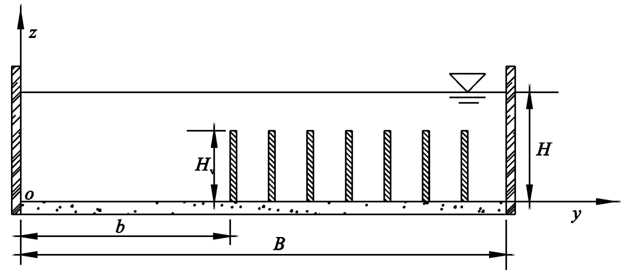

Fig. 4 Rectangular channel with submerged vegetated corner. y is the lateral coordinate, z is the coordinate normal to the bed, b is width of non-vegetated section, B is width of the channel, H_v is vegetation height, H is water depth

the system of linear equations. The lateral distribution of depth-average velocity in rectangular channel with submerged vegetated corner can be derived.

2.1.3 Case 3: symmetric two-stage rectangular channel with submerged vegetated floodplain

Compared with the former two cases, this case considers two-stage rectangular channel with submerged vegetation. The method required for this case synthesizes the methods of the former two cases. Based on this, an experiment in the two-stage rectangular channel with submerged vegetated floodplain is carried out for the present research.

For the symmetric two-stage rectangular channel with submerged vegetated floodplain (Fig. 1), the equations of the main channel and floodplain can be given, respectively, which are the same as equations (19) and (20), where the superscripts (1), (2) indicate the main channel and floodplain, respectively.

$$k_1 = \frac{8gS_0H}{f_1}\left(1 - \frac{\Gamma_1}{\rho g S_0 H}\right) \tag{34}$$

$$\gamma_1 = \sqrt{\frac{2}{\lambda_1}}\left(\frac{f_1}{8}\right)^{1/4}\frac{1}{H} \tag{35}$$

$$k_2 = \frac{gS_0(H-h) - \Gamma_2/\rho}{\frac{f_2}{8} + \frac{1}{2\delta}(C_D\beta A_v)H_v R_u^2} \tag{36}$$

$$\gamma_2 = \sqrt{\frac{2}{\lambda_2}}\left(\frac{8}{f_2}\right)^{1/4}\frac{\sqrt{\frac{f_2}{8} + \frac{1}{2\delta}(C_D\beta A_v)H_v R_u^2}}{H-h} \tag{37}$$

The appropriate boundary conditions were: (1) $\partial U_d^{(1)}/\partial y = 0$ at $y = 0$, (2) $\partial U_d^{(2)}/\partial y = 0$ at $y = B$, (3) $\alpha_u U_d^{(1)} = U_d^{(2)}$ at $y = b$, (4) $\partial U_d^{(1)}/\partial y = \partial U_d^{(2)}/\partial y$ at $y = b$. Applying these four boundary conditions to Equations (19) and (20), the coefficients A_1, A_2, A_3 and A_4 can be obtained, by solving the system of linear equations. Also, the lateral distribution of depth-average velocity in the symmetric two-stage rectangular channel with submerged vegetated floodplain could be derived.

2.2 Experimental data

An experiment was carried out for the two-stage rectangular channel with submerged vegetated floodplains as shown in Fig. 1. The experiment was conducted in the flume of Tsinghua University as shown in Fig. 5. The flume was 22.6 m long, 1.6 m wide, comprising a 0.4 m wide, 0.2 m deep main channel and two rigid 0.6 m wide floodplains, as shown in Fig. 6a. The bed slope of the flume was set to 0.001. The flume had three different water circulation systems: two internal ones, which re-circulated water from the downstream end, and one external one, which passed water through the flume to the main laboratory sump. Both sides of the flume were made of optical glass while the bed was made of cement. A straightener was set in flow access point of flume to homogenize the velocities of the cross-sections.

For a given discharge, the tailgate at the downstream end of the flume was adjusted to produce uniform flow conditions throughout the test length.

Vegetation was simulated by bamboo reeds, with the diameter (D) of 0.8 cm, the height (H_v) of 20 cm, and the vegetation spacing of ten centimeters. On each floodplain there were five lines of vegetation as shown in Fig. 6b.

a b

Fig. 5 Two-stage rectangular channel with submerged vegetated floodplain. (a) Top view. Both sides of the flume were made of optical glass while the bed was made of cement. Vegetation was simulated by bamboo reeds. (b) Side view. The vegetation was submerged

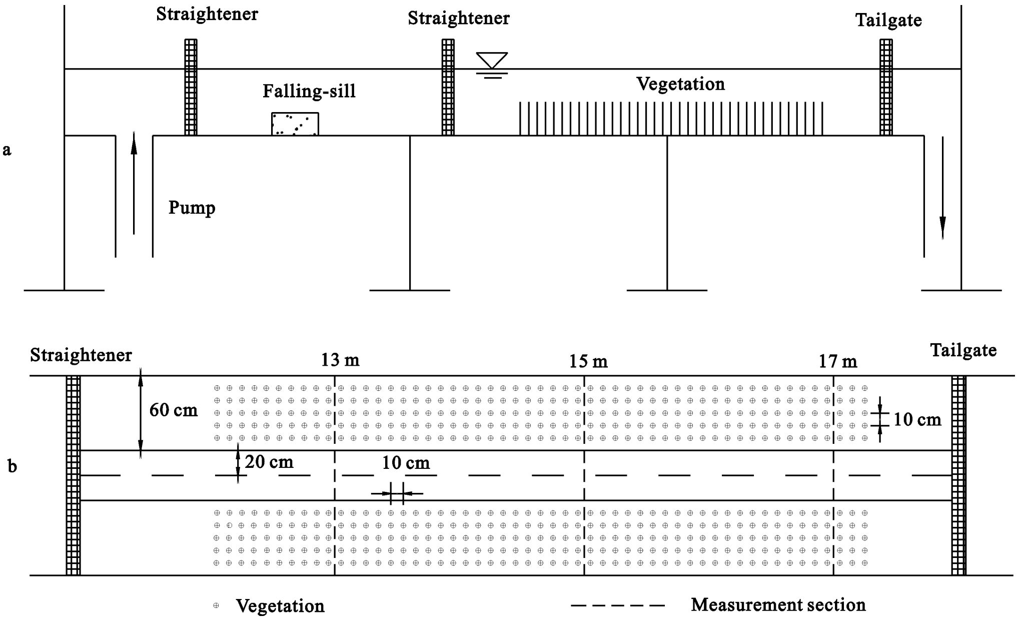

Fig. 6 Schematic description of flume. (a) Side view. Straighteners were set in flow access point of flume to homogenize the velocities of the cross-sections. The falling-sill was made of cement. The tailgate at the downstream end of the flume was adjusted to produce uniform flow conditions throughout the test length. (b) Top view. Vegetation spacing is 10 cm. Half of the main channel width is 20 cm. Floodplain width is 60 cm

Velocity was measured by electromagnetic current meter (JFE, Japan); the range was ±250 cm/s, the accuracy was ±2% or 0.5 cm/s and the sampling rate was 15 to 70 Hz.

In the experiment the sampling rate was set to 20 Hz; sampling time at each measurement-point was about 60–90 s. Every five centimeters set a vertical measuring line at each cross-section and every vertical line set 5–20 points. The measurement-points in one experiment case are shown in Fig. 7.

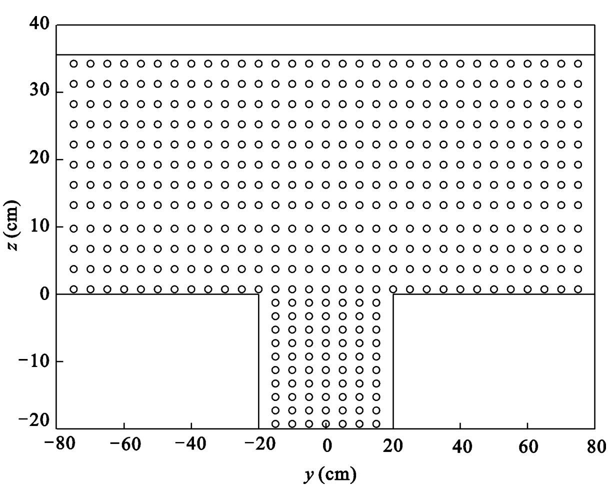

Fig. 7 Measurement-points of cross section. The circles denote measurement points

3 Results

3.1 Comparison of analytical velocity distributions with simulated results for Case 1

The mean flow and turbulence structures in compound open-channel flow with a vegetated floodplain were simulated by the developed Reynolds stress model (RSM) (Kang and Choi, 2006). In the computation, the water depth and channel width were the same as the previous case. The total width of the channel (B) is 0.4 m, the main channel width is 0.2 m, the water depth (H) is 0.08 m, the bankfull height (h) is 0.04 m. Vegetation densities A_v varied from 0.25/m to 4.00/m, where A_v is the projected area of the vegetation in the streamwise direction per unit volume. Since detailed measurement data were not included in the literature, the algebraic stress model (ASM) data (Naot *et al*., 1996) and RSM data (Kang and Choi, 2006) were used for comparison. The mean flow and turbulence structures in compound open-channel flows with emerged vegetation on the floodplain were simulated (Naot *et al*., 1996; Kang and Choi, 2006).

The lateral distributions of depth-averaged velocity computed by RSM and ASM are shown in Fig. 8. There are a difference between the RSM and ASM data for the floodplain.

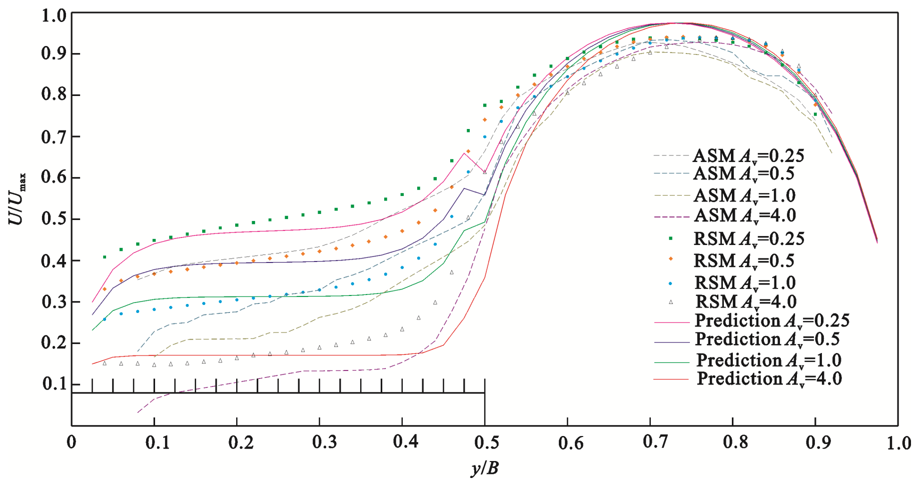

Fig. 8 Lateral distributions of depth-averaged velocity computed by Reynolds stress model (RSM), algebraic stress model (ASM) and analytical method

Comparison between the analytical results and RSM computational data demonstrated that there was an agreement between the RSM data and the analytical solutions. However, the comparison between the analytical results and ASM computational data showed that there was no agreement between the ASM data and the analytical solutions (shown in Fig. 8).

3.2 Comparison of analytical velocity distributions with measured results for Case 2

Measurements were taken in a wide (L = 6H) and partially vegetated open channel according to the experimental data (Naot *et al.*, 1996). The diameter of vegetation (D) is 0.06H. The experimental Case 1: nondimensional vegetation density (N) is 6, and shading factor for aligned vegetation (β) is 0.51; the experimental Case 2: nondimensional vegetation density (N) is 24, and shading factor for aligned vegetation (β) is 0.43; and the experimental Case 3: nondimensional vegetation density (N) is 96, and shading factor for aligned vegetation (β) is 0.26. Lateral distributions of depth-averaged velocity according to the experimental data are shown in Fig. 9. Since the experimental data for the vegetation corner were not given, the depth-averaged velocities were only obtained above the vegetation. Thus, the data in the vegetated section are depth-averaged velocities for the layer above the vegetation; and the data in the non-vegetated section are depth-averaged velocities over total flow depth.

The comparison between measured and analytical results is shown in Fig. 9. Based on the relation between the two depth-averaged velocities (1 – R_u), the analytical solutions above the vegetation layer were obtained from the analytical solutions over total flow depth. We can

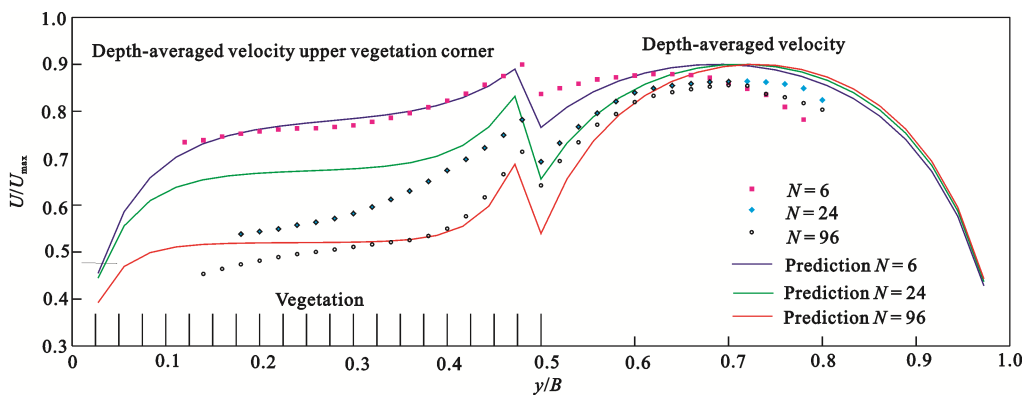

Fig. 9 Comparison between measured and analytical results. First three symbols denote experimental data in the legend; the next three lines denote analytical results. N is nondimensional vegetation density

infer that although the analytical solution results deviated a little from the experimental data as $N = 24$, other situations agreed very well as $N = 6$ and 96.

3.3 Comparison of analytical velocity distributions with experimental data for Case 3

The Case 3 experiment was carried out in the flume of Tsinghua University. Comparison between the measured and analytical result is shown in Fig. 10. For the two-stage rectangular compound open channel, which is different from the compound open channel with side wall, there was a discontinuous phenomenon of depth-averaged velocity at the stage point. The analytical solutions were more apparent with this phenomenon than the experimental data but they still agreed well with the experimental data.

4 Discussion

Based on the relation between the flow velocity U_r and U_T (Huthoff *et al.*, 2007), this paper assumes that U_v and U_d on the floodplain share a similar relation in Equation (5). However, there are many researches that predicted the vertical velocity distribution in an open channel flow with submerged vegetation. For example, a three-layer model was proposed to predict the vertical velocity distribution in an open channel flow with submerged vegetation (Huai *et al.*, 2009a). Since the equation was solved analytically, we adopted the relatively simple result of vertical velocity distribution (Huthoff *et al.*, 2007), which might have a certain effect on the prediction result. Furthermore, we applied different vertical velocity distributions for comparative analysis.

Since the equation was solved analytically, some variables in Equation (18) were considered as being constant. For instance, R_u was the same in the vegetated floodplain in this paper. This consideration might be a partial cause of disagreement between the prediction and experiment results, especially in the interface between vegetated area and non-vegetated area.

Velocity distribution of non-vegetated floodplain given by Liao and Knight (2007) was used to fill the gap of the boundary condition between the main channel and the floodplain in Equations (27) and (28). We consider that the change rates of U_d at the two sides were the same in Equation (28) as the assumption of Liao and Knight (2007). If the vegetation is not just located near the stage point on the floodplain, the velocity change rate should be the same as the main channel. If the vegetation were just located near the stage point on the floodplain, the velocity change rate should be totally different from that of the main channel. However, the vegetation being just located near the stage point on the floodplain is not like being located on the wall; there is a gap between two rods of vegetation. The assumption of Liao and Knight (2007) might be right for the gap. In general, the vegetated area is considered as a whole, the assumption might be advisable for these vegetated cases. However, the applicability of the assumption needs to be further studied. We reckon that the applicability of this assumption might be a partial cause of disagreement between prediction and experiment near the stage point on the floodplain. In this regard, the applicability of the four boundary conditions of velocities and velocity change rates in Case 2 as well as Case 1 needs to be studied further.

The differences between the experimental results and predictions at the stage points could be observed in Fig. 10 as well as in Fig. 8 and Fig. 9. The equations and the variables and the assumed boundary conditions in the three cases need further investigation, particularly the assumed boundary conditions at the stage points that are considered to contribute to the disagreement between the predicted and experimental data.

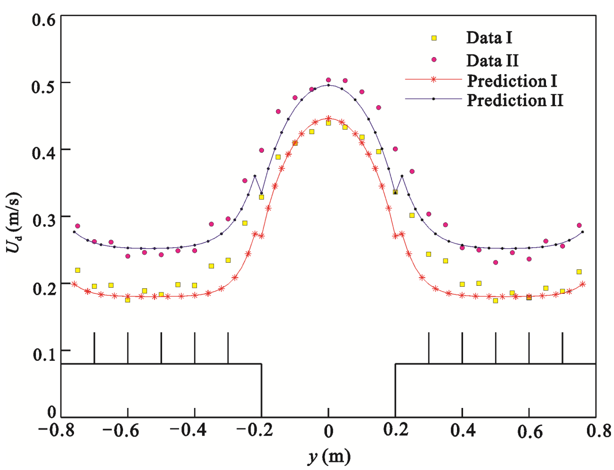

Fig. 10 Comparison between measured and analytical results. First two symbols denote experimental data. Experimental data I: water depth (H) is 49 cm; experimental data II: water depth (H) is 55 cm. The next two symbols are analytical results

5 Conclusions

Based on the analytical solution of the depth-integrated

Reynolds-Averaged Navier-Stokes equation with an additional term considering the effects of vegetation, the lateral distributions of depth-averaged velocity in open compound channels with emerged and submerged vegetated floodplains were analyzed and the following conclusions were made.

(1) There is a discontinuous phenomenon of depth-averaged velocity at the stage point of two-stage rectangular compound open channel. By applying the method for the depth-averaged velocity distributions in two-stage rectangular channels without vegetation, the authors extend to the two-stage rectangular channel with emerged vegetated floodplain.

(2) Due to the complexity of submerged vegetation, researches on the lateral distribution of depth-averaged velocity for submerged vegetated floodplain are limited. Based on the two-layer approach where flow above and through the vegetation layer was described separately, we have successfully obtained the lateral distribution of depth-averaged velocity in the main channel without vegetation and floodplain with submerged vegetation.

(3) Comprehensively considering the complexity of submerged vegetation and the discontinuous phenomenon of depth-averaged velocity at the stage point, we have successfully obtained the lateral distribution of depth-averaged velocity in the two-stage rectangular channel with submerged vegetated floodplain.

(4) The analytical solutions of the three cases of the open channels indicated that the corresponding analytical depth-averaged velocity distributions agree well with the simulated and experimental prediction.

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