

# Spatial Structure Characteristics Detecting of Landform based on Improved 3D Lacunarity Model

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**Abstract:** The spatial structure characteristics of landform are the foundation of geomorphologic classification and recognition. This paper proposed a new method on quantifying spatial structure characteristics of terrain surface based on improved 3D Lacunarity model. Lacunarity curve and its numerical integration are used in this model to improve traditional classification result that different morphological types may share the close value of indexes based on global statistical analysis. Experiments at four test areas with different landform types show that improved 3D Lacunarity model can effectively distinguish different morphological types per texture analysis. Higher sensitivity in distinguishing the tiny differences of texture characteristics of terrain surface shows that the quantification method by 3D Lacunarity model and its numerical integration presented in this paper could contribute to improving the accuracy of landform classifications and relative studies.

**Keywords:** digital elevation model (DEM); 3D Lacunarity model; spatial pattern; terrain texture; landform

**Citation:** Tao Yang, Tang Guo'an, Strobl Josef, 2012. Spatial structure characteristics detecting of landform based on improved 3D Lacunarity model. *Chinese Geographical Science*, 22(1): 88–96. doi: 10.1007/s11769-012-0516-2

## 1 Introduction

Terrain surfaces show complex morphological characteristics under the inner and outer geological forces. The quantification of spatial structure of terrain surface is not only the foundation of landform classifications, but also a key clue in revealing the rules of spatial differentiation of terrain surface configurations at the macro scale (Wilson and Gallant, 2000). Existing models and indices which are used to quantify the spatial structure of terrain surface mainly include statistical measures, information theory, and fractal geometry and so on. The shape characteristics of terrain surface are quantified by means of terrain factors in digital terrain analysis (Tang, 2000). Basic terrain factors such as slope, aspect and curvature portray the shape characteristics of terrain surface in different aspects. Among these terrain factors, surface roughness, variance coefficient of elevation and depth of surface cutting are applied to describe the ter-

rain characteristics in vertical direction of terrain surface. On the other hand, spatial structure of terrain surface are quantified usually by 2 dimensional landscape pattern analysis methods (Wu, 2000), which have been successfully applied in the study of terrain surface evaluation, landform classifications, soil erosion, as well as water and soil conservation (Wilson and Gallant, 2000; Hengl and Reuter, 2008). These quantification methods decompose 2.5 dimensional spatial structure characteristics to different directions of terrain surface. This process possesses relatively definite theoretical basis, but could not consider the 2.5 dimensional surface characteristics.

Above methods quantify the spatial structure characteristics by means of dimensionality reduction method and statistic value of terrain factor. These methods simplify the computation of quantification models. However, with regard to the quantitative indexes, similar values are usually obtained even to those terrains of dif-

Received date: 2010-12-14; accepted date: 2011-06-22

Foundation item: Under the auspices of National Natural Science Foundation of China (No. 40930531, 41171320, 41001301)

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ferent surface structure (Wu and Hobbs, 2007). Lacking of spatial structure information is the key reason for this phenomenon.

In recent years, with the enhancement of spatial data resolution and enrichment of imagery texture information, researches on image feature recognition and image classification based on textural property have gradually been active in fields of remote sensing analysis and computer vision (Kassim *et al.*, 2007; Backes *et al.*, 2010). Image texture is easily perceived by humans and believed to be a rich source of visual information about the nature and 3 dimensional shapes of physical objects (Materka and Strzelecki, 1998). Similar to image texture, spatial structures of terrain surface are also showed by terrain texture information based on digital elevation model (DEM). Therefore, this paper tries to quantify the spatial structure characteristics of terrain in virtue of texture analysis methods.

This paper attempts to present a new method on quantifying spatial structure characteristics of terrain surface based on the improved 3D Lacunarity model. Terrain texture is firstly enhanced by surface roughness model. Then improved 3D Lacunarity model is presented. Finally, the quantitative measures are experimentally evaluated and discussed by trial examinations.

This research could contribute to improving the reliability of landform classifications and other relevant studies.

## 2 Materials and Methods

### 2.1 Study area

Four test areas are selected in Shaanxi Province in China, which contains desert-loess transitional area, loess hilly and gully area, loess flat-topped ridge area in the Loess Plateau in the northern Shaanxi and middle mountain area in Qingling Mountains in the southern Shaanxi. These four test areas are distributed from north to south of Shaanxi, which stand for different landform types and show the varied morphological characteristics. The location of study areas in China and distribution of test areas are shown in Fig. 1.

### 2.2 Data source

DEM of four test areas are with grid size of 5 m, which are obtained from digitized contour of topographic map at a scale of 1 : 10 000. Range of test areas is 25 km<sup>2</sup> (5 km × 5 km). DEM data are provided by Shaanxi Geomatics Center of National Administration of Surveying, Mapping and Geoinformation (<http://www.sxgis.cn/>). The elevation (Fig. 2) and basic terrain parameters (Table 1)

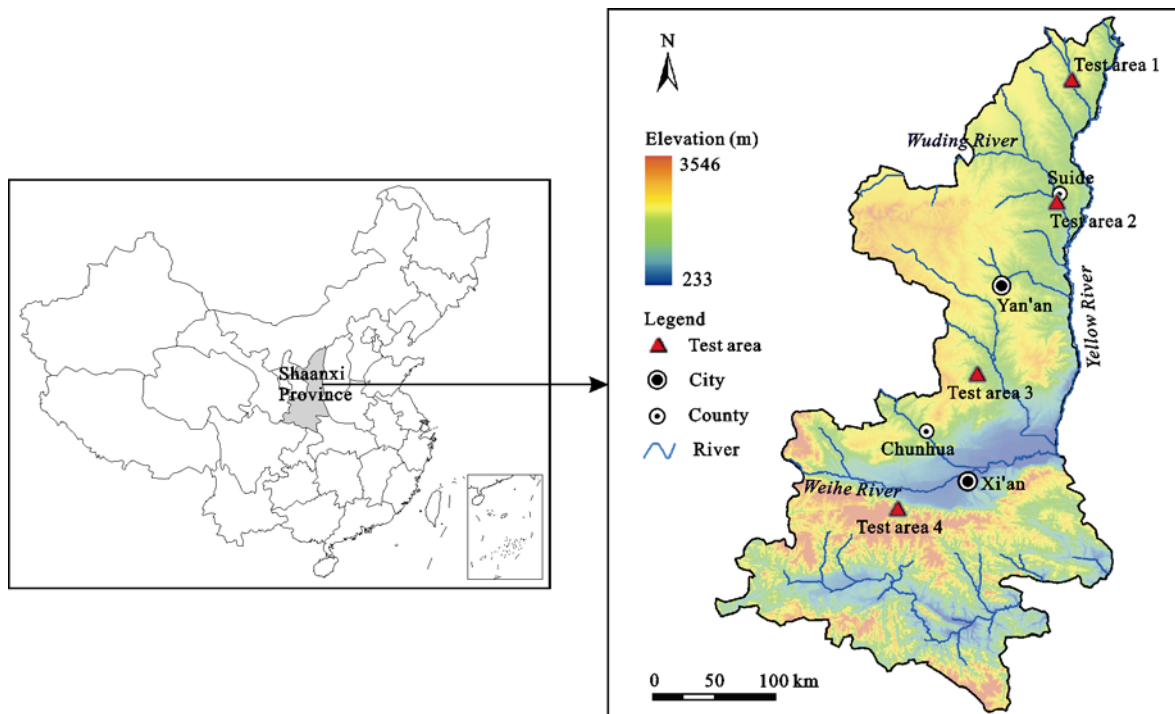


Fig. 1 Location of study area and distribution of test areas

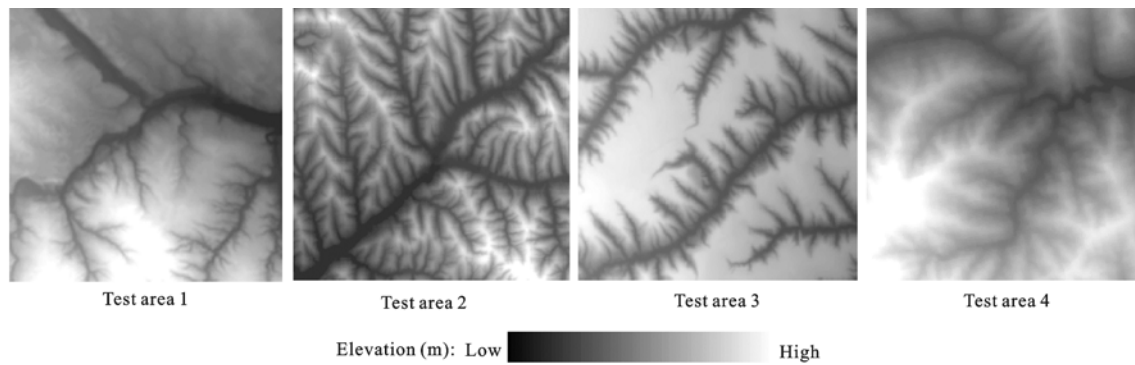


Fig. 2 DEM data of test areas

Table 1 Basic terrain parameters of test areas

Test area	Terrain type	Range of elevation (m)	Relative height difference (m)	Fractal dimension
Test area 1	Desert-loess transitional area	1025–1355	330	2.33
Test area 2	Loess hilly and gully area	836–1184	348	2.33
Test area 3	Loess flat-topped ridge area	778–1151	373	2.31
Test area 4	Middle mountain area	1520–2580	1060	2.30

of four test areas are as follows.

### 3 Method

#### 3.1 Spatial pattern modeling based on 3D Lacunarity model

Existing spatial pattern quantitative measurements based on image texture analysis mainly focus on two dimensional image information, including statistics models like gray level co-occurrence matrix (GLCM) and structure methods like Gabor filtering. In these measures, Lacunarity analysis method has more advantages in scale dependent measure of heterogeneity or texture (Gefen *et al.*, 1983) and spatial pattern analysis.

Lacunarity is regarded as a measure of the gappiness or hole-iness of a geometric structure (Kaye, 1989). As an index representing the terrain surface, Lacunarity was initially introduced by Mandelbrot (1983) for measuring the distribution of voids in a geometric object, with the object being more lacunar than others if its void sizes are distributed over a wider range. Unlike Lacunarity, the fractal dimension describes the texture of a fractal set. Two different fractal sets can have the same fractal dimension but a different Lacunarity. So a single fractal dimension alone is not enough to fully discriminate textures and natural surfaces. Mandelbrot proposed the concept of Lacunarity to discriminate textures (Akkari

*et al.*, 2009).

Lacunarity analysis method has two types of quantitative models: 2D-based Lacunarity model and 3D-based Lacunarity model. For 2D-based image texture analysis, Allain and Cloitre (1991) introduced a gliding-box algorithm for calculating Lacunarity and concluded that Lacunarity appears to be a new tool for characterizing the geometry of deterministic and random sets, and that the 'elusive notion' (Mandelbrot, 1983) of texture could be quantified by Lacunarity (Akkari *et al.*, 2009). In order to calculate the texture characteristics of grey scale image texture surface, Dong (2000) presented a 3D Lacunarity analysis model based on Differential Box Counting method which can provide more accurate texture measurements than some existing Lacunarity measures.

Compared with 2D image texture, terrain texture is presented by 3D-based terrain surface model (2.5D surface model exactly). The relief property based on elevation is the key factor that causes the variation of terrain texture. Thus, 3D-based Lacunarity model is used in this paper. The calculation method is as follows.

Based on the Differential Box Counting method (Sarkar and Chaudhuri, 1992), a cube of size  $r \times r \times r$  is placed over the upper left corner of an DEM with  $w \times w$  window size. Parameter  $w$  is an odd number, and  $r < w$ . Depending on the pixel values of DEM in the  $r \times r$  gliding box, a column with more than one cube may be

needed to cover the image intensity surface (Fig. 3). Assign numbers 1, 2, 3, ... to the cubic boxes from bottom to top. For each  $r \times r \times r$  gliding box, the box number containing minimum pixel values and maximum pixel values are denoted by min and max, respectively. Then the relative height of the column ( $n_r$ ) is calculated by Equation (1) (Dong, 2000).

$$n_r(i, j) = \max - \min + 1 \quad (1)$$

where  $i$  and  $j$  are coordinates of DEM. When the  $r \times r \times r$  gliding box moves throughout the  $w \times w$  analysis window, total numbers of gliding box ( $M_r$ ) are calculated by the following equation.

$$M_r = \sum_{i=1}^w \sum_{j=1}^w n_r(i, j) \quad (2)$$

The 3D Lacunarity model at scale  $r$  is defined by the mean-square deviation of the fluctuations of mass distribution probability  $Q(M_r, r)$  divided by its square mean (Allain and Cloitre, 1991):

$$A(r) = \frac{\sum M_r^2 Q(M_r, r)}{\sum M_r Q(M_r, r)} \quad (3)$$

where  $A(r)$  is the Lacunarity value.

As shown in Fig. 3, analysis window size is  $6 \times 6$ , cubic gliding box size is  $3 \times 3 \times 3$ . For the nine pixels in the upper left corner of DEM, the minimum and maximum pixel values are 2 and 8, which are located in the No. 1 and No. 3 gliding boxes, thus the minimum is 1 and maximum is 3.

In order to test the validity of 3D Lacunarity model and reduce the uncertainty which is caused by matching process between cubic window size ( $r$ ) and analysis window size ( $w$ ), cubic gliding box size is set by  $3 \times 3 \times 3$ .

3D-based Lacunarity model describes the spatial pattern characteristics of terrain surface in analysis scale  $r$ . Under the suitable analysis scale, the variation between analysis windows in complicated and strong relief of terrain surface is larger than plain terrain, which corresponding to larger Lacunarity value. In ideal plain terrain, the Lacunarity value  $A(r)$  is 1.

Lacunarity model is a scale dependent measure of heterogeneity which means Lacunarity is the function of analysis window size  $r$ . By means of changing the window size  $r$ , the correlation between Lacunarity value

and analysis window size is shown by curve  $C_l$  and its natural logarithmic curve  $C_l$ . In order to quantify the characteristics of curves, numerical integration ( $L$ ) of  $C_l$  is calculated. At last, the spatial pattern of terrain surface can be described as:

$$\Omega = F(\lambda, C_l, C_l, L) \quad (4)$$

where  $\Omega$  is the Lacunarity value;  $\lambda$  is the analysis function between terrain surface data and 3D-based Lacunarity model. If DEM data is used in  $\Omega$  directly,  $\lambda$  is null.

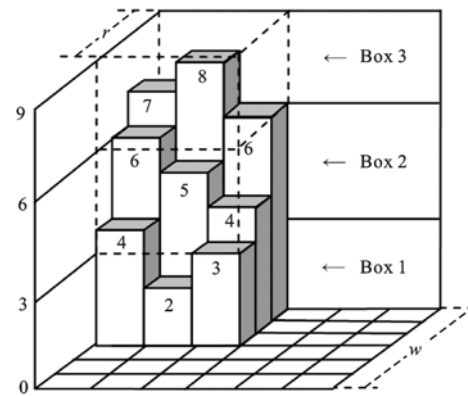


Fig. 3 3D Lacunarity model based on Differential Box Counting method (Referencing to Dong, 2000)

### 3.2 Enhancement of DEM structure characteristics

Texture of terrain surface belongs to natural texture which has no regular structure strictly, especially in plain area, where the surface roughness is lower and terrain texture is not distinct. In order to get the spatial pattern characteristics of terrain surface, texture of terrain surface should be enhanced before applying the Lacunarity model.

In local analysis window of DEMs, surface texture are shown by vertical structure characteristics which are described by those terrain factors such as slope, curvature, roughness (Tang, 2000). In these terrain factors, surface roughness is widely used in quantifying the complex of terrain in local scale (Day, 1979; Beasom, 1983; Berry, 1993; Frankel and Dolan, 2007; Grohmann et al., 2009). Hobson (1972) presented a method to calculate the surface roughness by the ratio of real surface area to its projection area of square cells. Jenness (2004) presented a straightforward method to calculate surface area grids directly from DEM which is applied in this paper to calculate surface roughness value.

As shown in Fig. 4a, the real area of central grid

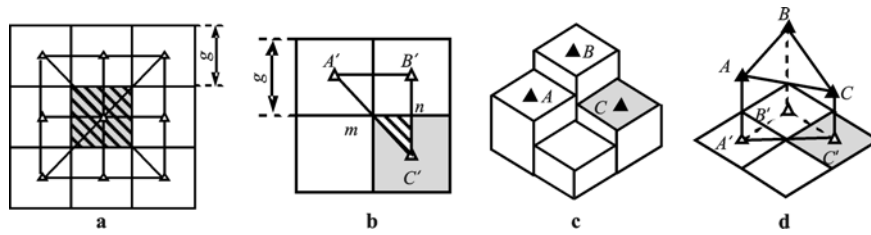


Fig. 4 Sketch maps of spatial area calculation of grid-based DEM (Referencing to Jenness, 2004)

(shadow grid) is calculated by eight surrounding grids. The upper left four grids and their spatial model are shown in Fig. 4b and Fig. 4c. The grid size of DEM is  $g$ , elevation of grid A and B is  $h_A$  and  $h_B$ , thus the real area of spatial triangle  $\triangle ABC$  can be calculated by Equation (5).

$$S_{\triangle ABC} = \sqrt{q(q-l_{AC})(q-l_{AB})(q-l_{BC})} \quad (5)$$

where  $q$  is the average length of three spatial lines that made the spatial triangle  $\triangle ABC$ , the length of spatial line  $l_{AB}$  can be calculated by Equation (6).

$$l_{AB} = \sqrt{g^2 + (h_B - h_A)^2} \quad (6)$$

The real area of shadow grid can be calculated by eight spatial triangles which connecting each cell center point with the center points of the eight surrounding grids.

At last, surface roughness ( $R_r$ ) can be calculated by Equation (7).

$$R_r = \frac{S_r}{S_t} \quad (7)$$

where  $S_r$  is the spatial surface area and  $S_t$  is the projection area of grid.

### 3.3 Improved 3D Lacunarity model

In this paper, surface roughness ( $R_r$ ) is set as the analysis function  $\lambda$  in Equation (4). This process can fuse the vertical structure information of terrain surface into spatial pattern analysis. So the spatial pattern of terrain surface can be improved as:

$$\Omega_t = F(R_r, C_t, C_l, L) \quad (8)$$

where  $\Omega_t$  is the improved Lacunarity value;  $C_t$  and  $C_l$  are the correlation curve of Lacunarity value against analysis window size and its natural logarithmic curve;  $L$  is the numerical integration of  $C_l$ .

The improved 3D Lacunarity model described above firstly enhances the vertical characteristic by surface roughness model based on area ratio model (Grohmann *et al.*, 2009), especially enhances the texture structure of

micro relief terrain such as plain terrain. Then spatial pattern characteristics can be quantified by 3D-based Lacunarity curves and its numerical integration value.

## 4 Results and Analyses

### 4.1 Texture enhancement of landform

The texture information of terrain surface describes the spatial structure characteristics of terrain in macro scale, which are shown by the variation of terrain elevation information.

The results of texture enhancement based on the improved 3D Lacunarity model are shown in Fig. 5. Compared with original gray level based on DEM images, spatial structure characteristics of landform have been enhanced, especially in gully and low relief area like the top-flat area of loess tableland. By means of texture enhancement of landform, spatial structure characteristics of landform are much easier to be detected by texture analysis models.

In order to detect the sensitivity of local terrain variation based on surface roughness  $R_r$ , this paper probes into the matching effects between profile characteristics of DEM and surface roughness. Surface roughness model is calculated by a  $3 \times 3$  neighborhood window, which is caused by the surrounded eight neighborhood grids. So the profile characteristics of DEM depend on three profile lines (Fig. 6), one DEM profile and two neighbor profiles on the two sides of DEM profile. This profile is selected from test area 3. The grid size of DEM data and surface roughness  $R_r$  are both 5 m. As shown in Fig. 6, surface roughness  $R_r$  is more sensitive to the variation of relief, especially at the location with obvious slope transition and big gaps in DEM profile and two neighbor profiles.

However, due to limited range of values of  $R_r$ , the statistical values of  $R_r$  in different test areas are not significant (Table 2). It seems very difficult to distinguish

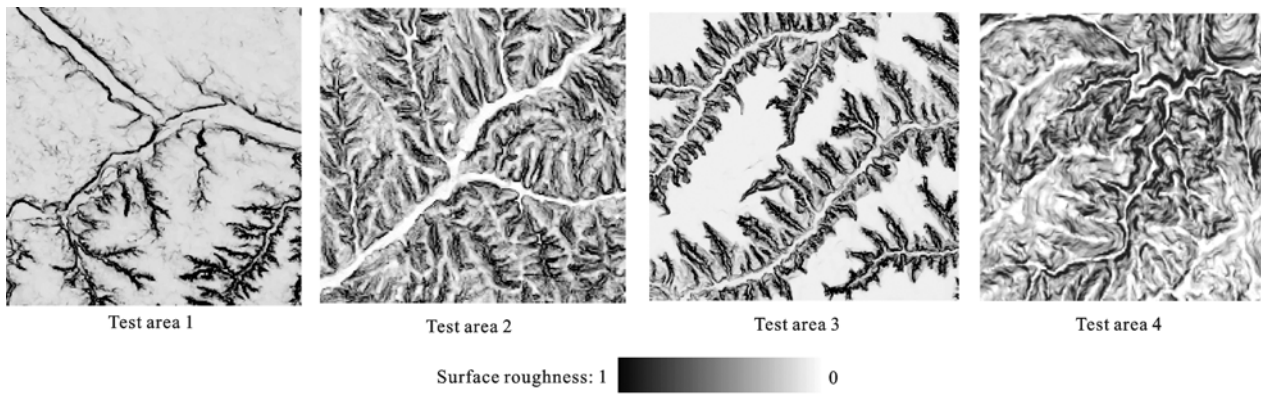


Fig. 5 Surface roughness of test areas

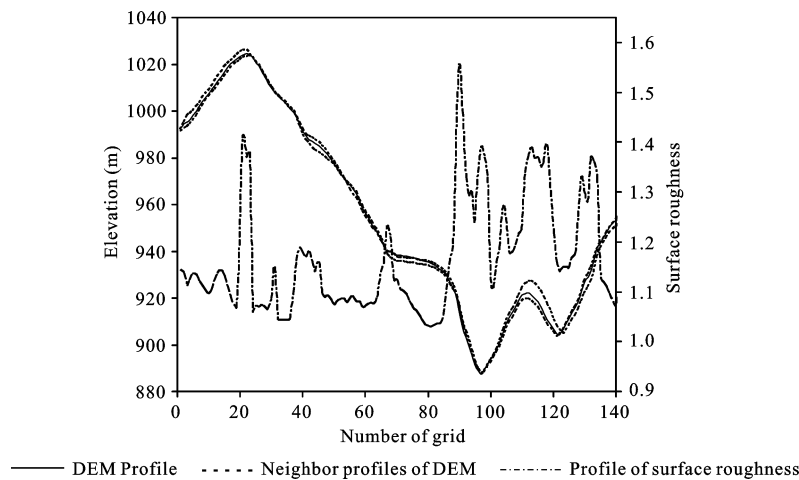


Fig. 6 Matching results between surface roughness and DEM profiles

Table 2 Statistics of surface roughness ( $R_r$ ) in test areas

Test area	$R_r$		
	Maximum	Average	Standard deviation
1	1.61	1.03	0.07
2	2.32	1.20	0.16
3	2.98	1.14	0.21
4	3.82	1.29	0.20

different types of landforms only by global statistical values. So how to design a useful model or index which could effectively detect and distinguish different types of spatial structure characteristics is the key problem should be considered.

#### 4.2 Spatial structure analysis based on improved 3D Lacunarity model

3D-based Lacunarity model is a multi-scaled method of determining the texture associated with patterns of spatial dispersion (Plotnick *et al.*, 1993). In order to calcu-

late the numerical integration  $L$  of  $C_l$ , the right boundary of analysis windows range in curve  $C_l$  which is denoted by  $w_{\max}$  need to be determined firstly. With the increase of analysis window size ( $w$ ), the spatial heterogeneity among adjacent analysis windows decreases gradually. When the Lacunarity value  $A(r)$  of curve  $C_l$  approaches to 1, the variation of adjacent analysis windows can be regarded as no difference and the calculation process is end.

In ideal condition, the Lacunarity value of curve  $C_l$  is converging to 1. However, because of the wide range and much complexity of test areas in terrain analysis, the Lacunarity value of curve  $C_l$  is impossible to converge to 1 in practical applications. 0.001 is set as the convergence threshold of the Lacunarity value in this experiment, namely, if the Lacunarity value of  $C_l$  is less than 0.001 with increase of analysis window size, the calculation process will be end. The maximum analysis window size ( $w_{\max}$ ) of four test areas are  $149 \times 149$ ,  $191 \times$

191,  $139 \times 139$  and  $167 \times 167$ , respectively.

The correlation curves ( $C_l$ ) between Lacunarity value and analysis window size are shown in Fig. 7 and its natural logarithmic curves ( $C_l$ ) are shown in Fig. 8. When the analysis window size  $w$  is smaller, the analysis window is either completely occupied by the texture features of terrain surface or a unique value. In this period, the variation of adjacent analysis windows is higher and the variation inside of analysis window is lower in which the Lacunarity value is higher. With the increase of analysis window size, the detailed texture features may be less than the size of analysis window. Thus, the proportion of texture features in analysis window would be gradually close to the non-texture features. The variation of adjacent analysis windows would decrease gradually, so does the Lacunarity values. The curves of  $C_l$  and  $C_l$  show a trend of decrease with the increase of analysis window size. As the analysis window size  $w$  is getting close to the maximum analysis window size, the Lacunarity value of curve  $C_l$  and  $C_l$  trend to converge to 1 and 0, respectively.

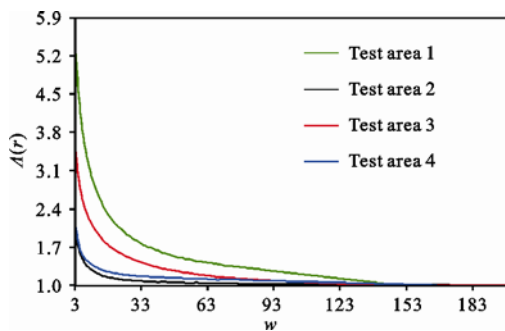


Fig. 7 Correlation curve ( $C_l$ ) between Lacunarity value  $A(r)$  and analysis window size ( $w$ ) based on 3D Lacunarity model

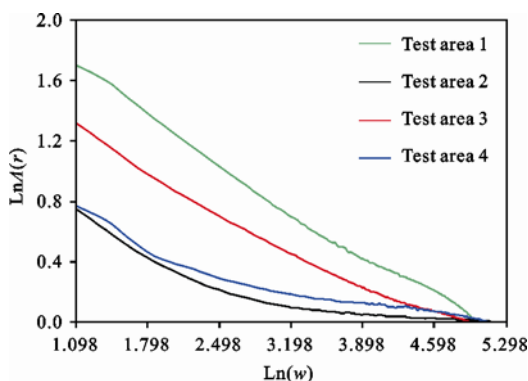


Fig. 8 Natural Logarithmic curve ( $C_l$ ) between Lacunarity value  $A(r)$  and analysis window size ( $w$ )

Four test areas stand for four classic landform types in Shaanxi Province. Test area 1 is selected from desert-loess transitional areas which is covered by thin sand dunes. The overall trend of terrain surface is relatively flat with several gullies which reflect the spatial structure characteristics. Due to the significant distribution of the gully with large flat areas surrounded, the texture characteristics are obvious and the Lacunarity value is the highest in these four areas. Compared with test area 1, the density of gullies is higher and the area of gaps between gullies is relatively lower in test area 3. Thus, the Lacunarity value of test area 3 is lower than test area 1 (Fig. 7 and Fig. 8).

Compared with desert-loess transitional area and loess flat-topped ridge area, loess hilly and gully area show different spatial structure characteristics. Due to intense erosion, more gullies are generated in these test areas. The gap area between gullies in loess hilly and gully area is so smaller than that in the loess flat-topped ridge area that the Lacunarity value is lower. What's more, the spatial structure of test area 2 is similar to the regular distribution pattern. So the curve  $C_l$  is trend to the characteristic curve of ideal regular distribution (Dong, 2000). The texture of middle mountain area is similar with loess hilly and gully area except for higher relative height difference. So the Lacunarity curve of middle mountain area is close to that in loess hilly and gully area.

Numerical integration  $L$  of curve  $C_l$  for four test areas are 3.1955, 0.8162, 2.1438 and 1.0628, respectively. Compared with the fractal dimension (Table 1), the numerical integration  $L$  effectively describes the variation of different texture features. As a quantitative index, the numerical integration  $L$  matches the trend of Lacunarity curves.

However, test areas 2 and 4 have different relative height difference but similar gully density and spatial distribution, so the numerical integration of Lacunarity curve  $C_l$  is close. In these areas with similar texture features of terrain surface, other topographic factors containing vertical relief information need to be considered in the pattern recognition of terrain features, so as to improve the result of landform recognition and classification.

### 4.3 Scale domain of 3D Lacunarity model

The results of 3D Lacunarity model indicate that 3D

Lacunarity model is a scale dependent measurement. The pioneering works prove that the appropriate scale can be detected by the goodness of linear fit ( $R^2$ ) to Lacunarity curve  $C_l$  (Plotnick *et al.*, 1993; 1996). If the trend of the goodness of linear fit ( $R^2$ ) to Lacunarity curve  $C_l$  is a linear or nearly linear variation, the analysis window size could be regarded as the appropriate scale domain.

This paper detects the scale domain of Lacunarity model by three spans of analysis windows ( $3 \times 3$ ,  $5 \times 5$ ,  $7 \times 7$ , or  $11 \times 11$ ,  $13 \times 13$ ,  $15 \times 15$ ). In terms of linear fitting function, the goodness of linear fit ( $R^2$ ) against analysis windows is calculated and the results are shown in Fig. 9.

The analysis range of four test areas is nearly the same ( $5 \text{ km} \times 5 \text{ km}$ ). The result of the goodness of linear fit ( $R^2$ ) to Lacunarity curve  $C_l$  indicates that the maximum appropriate analysis window size of four test areas are  $21 \times 21$ ,  $29 \times 29$ ,  $21 \times 21$  and  $23 \times 23$ , respectively. If the analysis window size exceeds this maximum value, the fitting curve would show the ripple phenomenon significantly. Therefore, the scale between the minimum analysis window size ( $3 \times 3$ ) and the maximum appropriate analysis window size is the stable scale domain of test area in Lacunarity model.

## 5 Conclusions

Texture analysis method based on improved 3D Lacunarity model was employed in this paper to investigate the spatial structure quantifying of terrain surface. The main conclusions from experimental results are as follows.

(1) Surface roughness model is more sensitive to the

variation of relief, especially at the location with obvious slope transition and big gaps in DEM profile and two neighbor profiles of DEM. So, surface roughness model could effectively enhance the texture characteristics of terrain surface.

(2) In virtue of Lacunarity curve and its numerical integration, different types of terrain can be distinguished based on the variation of texture characteristics of terrain surface. Compared with traditional statistical indexes, like fractal dimension, the improved 3D Lacunarity model presented in this paper can effectively improve the classification and recognition accuracy of landform characteristics in the Loess Plateau.

(3) 3D Lacunarity model is not sensitive to the boundary of test areas and not require the stationary hypothesis of test data which improves the applicability of 3D Lacunarity model in DEM based on terrain analysis. In order to reduce the uncertainty which is caused in the matching process between cubic window size ( $r$ ) and analysis window size ( $w$ ), cubic gliding box size is set by  $3 \times 3 \times 3$ .

The experimental results show that it is difficult to distinguish the terrain types with similar texture characteristics of terrain surface only by the index of Lacunarity value, any other topographic factors that effectively reflect the vertical relief characteristics need to be considered in pattern recognition of terrain features to improve the accuracy of recognition and classification of landform.

## Acknowledgement

The authors acknowledge the financial support provided by the PhD Scholarship from Eurasic Pacific Uninet for

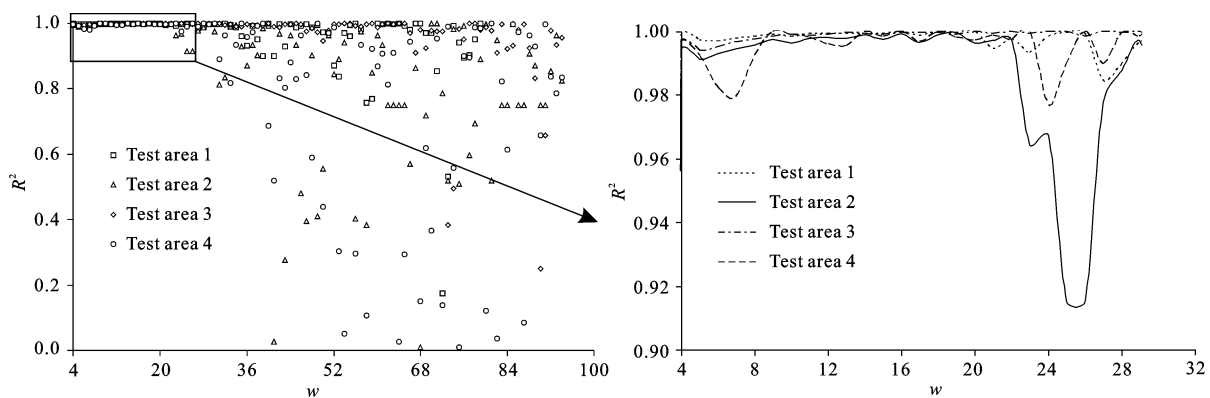


Fig. 9 Scale characteristics of goodness of linear fit ( $R^2$ ) to Lacunarity logarithmic curve ( $C_l$ )



collaboration research in Austria. We also thank for good suggestions and help from Dr. Lucian Drăguț and Clemens Eisank in University of Salzburg.

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