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Perturbed Solving Method for Interdecadal Sea-air Oscillator Model

MO Jiaqi[1, 2], LIN Yihua[3], LIN Wantao[3], CHEN Lihua[4]

(*1. Department of Mathematics, Anhui Normal University, Wuhu 241003, China; 2. Division of Computational Science, E-Institutes of Shanghai Universities at Shanghai Jiaotong University, Shanghai 200240, China; 3. State Key Laboratory of Numerical Modeling for Atmospheric Sciencens and Geophysical Fluid Dynamic, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China; 4. Department of Mathematics and Computer Science, Fuqing Branch of Fujian Normal University, Fuqing 350300, China*)

Abstract: A coupled system of the interdecadal sea-air oscillator model is studied. The El Niño-southern oscillation (ENSO) atmospheric physics oscillation is an abnormal phenomenon involved in the tropical Pacific ocean-atmosphere interactions. The oscillator model is involved with the variations of both the eastern and western Pacific anomaly patterns. This paper proposes an ENSO atmospheric physics model using a method of the perturbation theory. The aim is to create an asymptotic solving method for the ENSO model. Employing the perturbed method, the asymptotic solution of corresponding problem is obtained, and the asymptotic behaviour of the solution is studied. Thus we can obtain the prognoses of the sea surface temperature anomaly and related physical quantities.

Keywords: nonlinear equation; perturbation; El Niño-southern oscillator model; interdecadal sea-air oscillator

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1 Introduction

Interactions between the ocean and atmosphere contribute to climate fluctuations over a broad spectrum of time scales. Studies on those interactions have focused on El Niño-southern oscillation (ENSO) phenomena that have a period of 3−4 years and whose principal signature is in the tropical Pacific. Superimpositions on this natural mode of the coupled ocean-atmosphere system are interdecadal fluctuations that contribute to the irregularity of the southern oscillation. The southern oscillation involves an east-west redistribution of warm surface waters. In the eastern equatorial tropical Pacific a shoaling of subsurface isotherms signals the end of El Niño and the return of colder surface waters.

The ENSO is an interannual phenomenon involved in the tropical Pacific ocean-atmosphere interactions, and a very attractive object of study in the international academic circles. Many scholars have investigated the circulation in the upper Pacific (McPhaden and Zhang, 2002), the decadal climate variability (Biondi *et al.*, 2001), the interdecadal climate variability (Gu and Philander, 1997; Wang *et al.*, 1999), the ENSO mechanisms (Wang, 2001; Kushnir *et al.*, 2002), the air-sea coupled model in the tropics (Lin and Fu, 2001), the evolution equations (Lin *et al.*, 2002) and the instability evolution of air-sea oscillator (Feng *et al.*, 2002).

Some researchers considered the approximate solutions for a class of problems in atmospheric physics, solving method in equator Pacific (Mo *et al.*, 2006), the singularly perturbed solution of coupled model (Mo *et al.*, 2008), the global climate model (Mo *et al.*, 2009a;

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Correseponding author: MO Jiaqi. E-mail: mojiaqi@mail.ahnu.edu.cn

2009b), the approximate solution for wind-driven ocean circulation (Mo *et al*., 2010), and the dissipative travelling wave solution for ENSO (Wen *et al*., 2010).

The essence of ENSO oscillation requires both positive and negative ocean-atmosphere feedbacks. This anomaly reduces the zonal sea surface temperature (SST) gradient and hence the strength of the southern oscillation circulation, resulting in weaker trade winds around the equator. The weaker trade winds in turn drive the ocean circulation changes, thereby reinforcing SST anomaly. This positive ocean-atmosphere feedback or coupled ocean-atmosphere instability leads the equatorial Pacific to a never-ending warm state.

Recently, many scholars have investigated the nonlinear problem (Sagon, 2008; Barbu and Cosma, 2009; Ramos, 2009). Approximate methods have been refined, including the method of averaging, the matched asymptotic expansion method, the boundary layer method, and the multiple scales method. In this paper, we consider a sea-air oscillator model using a simple and valid method of the perturbation theory.

The response of the atmosphere to the warming in the tropics involves an intensification of the extratropical westerlies, leading to colder surface waters in extratropical regions that happen to be windows to the equatorial thermocline. It implys a continual, interdecadal climate fluctuation with a period that depends on the time it takes for water parcels to travel from the extratropics to the equator.

2 A Sea-air Oscillator Model

The coupled system for a sea-air oscillator time delay model of interdecadal climate fluctuations (Gu and Philander, 1997) is

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \alpha\gamma(T_2 - T_1) - \delta_1 T_2(t - d_2) + \lambda_1 T_1 - \varepsilon P_1(T_1) \quad (1)$$

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \gamma(T_2 - T_1) - \delta_2 T_1(t - d_1) + \lambda_2 T_2 - \varepsilon P_2(T_2) \quad (2)$$

where T_1 is the extratropical temperature at approximate region 25°N to 50°N (and 25°S to 50°S); T_2 is the tropical temperature at approximate region 20°S to 20°N; t denotes the time; α is the fraction of poleward atmospheric heat transport that remains in the extratropical regions; λ_1 is the negative constant; λ_2 is the feedback parameter; γ, δ_1 and δ_2 are positive constants; d_1 and d_2 are the delay time and $\lim_{\|T_i\| \to 0} \|P_i(T_i)\| = 0 \ (i = 1, 2)$ represent stochastic forcing from weather systems unrelated to tropical temperature variations for T_1 and T_2, respectively. In this paper, we assume that $\varepsilon = d_1 = d_2$ are the small positive parameters.

We first consider $\varepsilon = d_1 = d_2 = 0$ and corresponding linear equations (1) and (2):

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = (\lambda_1 - \alpha\gamma)T_1 + (\alpha\gamma - \delta_1)T_2$$

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = -(\gamma + \delta_2)T_1 + (\lambda_2 + \gamma)T_2$$

It is easy to see that the characteristic roots are

$$r_{1,2} = \frac{1}{2}[(\lambda_1 + \lambda_2 + (1-\alpha)\gamma] \pm$$
$$\sqrt{(\lambda_1 + \lambda_2 + (1-\alpha)\gamma)^2 - 4[(\alpha\gamma - \delta_1)(\gamma + \delta_2)]}$$

From the fraction of poleward atmospheric heat transport $\lambda_1 + \lambda_2 > (\alpha - 1)\gamma$ and $\alpha\gamma > \delta_1$, we know that the characteristic roots r_1 and r_2 possess different positive real parts. Thus the zero point for the linear system is unstable.

From the nonlinear systems (1) and (2), we have

$$\lim_{\|T_1\|,\ \|T_2\| \to 0} \frac{\varepsilon(P_1^2 + P_2^2)^{1/2}}{(T_1^2 + T_2^2)^{1/2}} = 0$$

Then the singular point of the nonlinear systems (1) and (2) is unstable, too. So the path curves on the phase plane for the systems (1) and (2) are away from the origin.

3 Perturbation Solution

Let

$$T_j(t) = \sum_{i=0}^{\infty} T_{ji}(t)\varepsilon^i, \quad j = 1, 2 \quad (3)$$

And developing the time delay functions $T_j(t-\varepsilon)$ in ε:

$$T_j(t-\varepsilon) = T_j(t) - \frac{\mathrm{d}T_j(t)}{\mathrm{d}t}\varepsilon + \frac{1}{2}\frac{\mathrm{d}^2 T_j(t)}{\mathrm{d}t^2}\varepsilon^2 + \cdots + \frac{(-1)^i}{i!}\frac{\mathrm{d}^i T_j(t)}{\mathrm{d}t^i}\varepsilon^i + \cdots, \quad j = 1, 2 \quad (4)$$

Substituting equations (3) and (4) into equations (1) and (2), developing nonlinear terms in ε, equating coefficients of the same powers of ε at both two sides of the equations respectively, for $i = 1, 2, \ldots$, we obtain

$$\frac{\mathrm{d}T_{10}}{\mathrm{d}t} = (\lambda_1 - \alpha\gamma)T_{10} + (\alpha\gamma - \delta_1)T_{20} \quad (5)$$

$$\frac{\mathrm{d}T_{20}}{\mathrm{d}t} = -(\gamma + \delta_2)T_{10} + (\lambda_2 + \gamma)T_{20} \quad (6)$$

$$\frac{\mathrm{d}T_{1i}}{\mathrm{d}t} = (\lambda_1 - \alpha\gamma)T_{1i} + (\alpha\gamma - \delta_1)T_{2i} + F_{1i} \quad (7)$$

$$\frac{\mathrm{d}T_{2i}}{\mathrm{d}t} = -(\gamma + \delta_2)T_{1i} + (\lambda_2 + \gamma)T_{2i} + F_{2i} \quad (8)$$

where F_{ji} ($j = 1, 2$; $i = 1, 2, \ldots$) are determined functions whose constructions are omitted.

It is easy to see that the general solution $T_{j0}(t)$ of the systems (5) and (6) is

$$T_{j0}(t) = C_j \exp(r_1 t) + D_j \exp(r_2 t), \quad j = 1, 2 \quad (9)$$

where r_j ($j = 1, 2$), are characteristic roots of systems (5) and (6):

$$r_{1,2} = \frac{1}{2}[\lambda_1 + \lambda_2 + (1-\alpha)\gamma] \pm \sqrt{\Delta}] \quad (10)$$

$$\Delta = (\lambda_1 + \lambda_2 + (1-\alpha)\gamma)^2 - 4[(\alpha\gamma - \delta_1)(\gamma + \delta_2)] \quad (11)$$

C_1, D_1 are arbitrary constants, and

$$C_2 = (r_1 + \alpha\gamma - \lambda_1)C_1 / (\alpha\gamma - \delta_1)$$

$$D_2 = (r_2 + \alpha\gamma - \lambda_1)D_1 / (\alpha\gamma - \delta_1)$$

From equations (7) and (8) for $i = 1$, we have

$$\frac{\mathrm{d}T_{11}}{\mathrm{d}t} = (\lambda_1 - \alpha\gamma)T_{11} + (\alpha\gamma - \delta_1)T_{21} + \delta_1 T_{20}' + P_1(T_{10}) \quad (12)$$

$$\frac{\mathrm{d}T_{21}}{\mathrm{d}t} = -(\gamma + \delta_2)T_{11} + (\lambda_2 + \gamma)T_{21} + \delta_2 T_{10}' + P_2(T_{20}) \quad (13)$$

We can obtain a set of solution T_{j1} $(j = 1, 2)$ for the systems (12) and (13) with zero initial conditions that

$$T_{11}(t) = -\frac{1}{\sqrt{\Delta}}\int_0^t [[(r_2 + \alpha\gamma - \lambda_1)(\delta_1 T_{20}'(\tau) + P_1(T_{10}(\tau))) - \frac{C_1}{D_1}(r_1 + \alpha\gamma - \lambda_1)(\delta_2 T_{10}'(\tau) + P_2(T_{10}(\tau)))] \exp(\gamma_1(t-\tau)) + [(r_1 + \alpha\gamma - \lambda_1)(-(\delta_1 T_{20}'(\tau) + P_1(T_{10}(\tau)) + \frac{C_1}{D_1}(\delta_2 T_{10}'(\tau) + P_2(T_{10}(\tau)))] \exp(\gamma_2(t-\tau))]\mathrm{d}\tau \quad (14)$$

$$T_{21}(t) = -\frac{1}{\sqrt{\Delta}}\int_0^t [[\frac{D_1}{C_1}(r_2 + \alpha\gamma - \lambda_1)(\delta_1 T_{20}'(\tau) + P_1(T_{10}(\tau))) + (r_1 + \alpha\gamma - \lambda_1)(\delta_2 T_{10}'(\tau) + P_2(T_{10}(\tau)))]\exp(\gamma_1(t-\tau)) + [(r_2 + \alpha\gamma - \lambda_1)(-\frac{D_1}{C_1}(\lambda_1 T_{10}'(\tau) + P_1(T_{10}(\tau))) + (\lambda_2 T_{20}'(\tau) + P_2(T_{20}(\tau))))]\exp(\gamma_2(t-\tau))]\mathrm{d}\tau \quad (15)$$

We can also obtain a set of solution T_{ji} ($j = 1, 2$; $i = 2, 3, \ldots$) for the systems (12) and (13) with zero initial conditions that

$$T_{1i}(t) = -\frac{1}{\sqrt{\Delta}}\int_0^t [[(r_2 + \alpha\gamma - \lambda_1)F_{1i} - \frac{D_1}{C_1}(r_1 + \alpha\gamma - \lambda_1)F_{2i}] \exp(\gamma_1(t-\tau)) + [(r_1 + \alpha\gamma - \lambda_1)(-F_{1i} + \frac{C_1}{D_1}F_{2i})] \exp(\gamma_2(t-\tau))]\mathrm{d}\tau \quad (16)$$

$$T_{21}(t) = -\frac{1}{\sqrt{\Delta}}\int_0^t [[\frac{D_1}{C_1}(r_2 + \alpha\gamma - \lambda_1)F_{1i} + (r_1 + \alpha\gamma - \lambda_1)F_{2i}] \exp(\gamma_1(t-\tau)) + [(r_2 + \alpha\gamma - \lambda_1)(-\frac{D_1}{C_1}F_{1i} + F_{2i})] \exp(\gamma_2(t-\tau))]\mathrm{d}\tau \quad (17)$$

Thus from equations (9), and (14)−(17), we have a set of the nth order approximation solution of the coupled systems (1) and (2) for a sea-air oscillator model:

$$T_1(t) = C_i \exp(r_1 t) + D_i \exp(r_2 t) - \frac{1}{\sqrt{\Delta}}\int_0^t [[\varepsilon[(r_2 + \alpha\gamma - \lambda_1)(\delta_1 T_{20}'(\tau) + P_1(T_{10}(\tau))) - \frac{C_1}{D_1}(r_1 + \alpha\gamma - \lambda_1)(\delta_2 T_{10}'(\tau) + P_2(T_{10}(\tau)))] + \sum_{i=2}^{n} F_{1i}(\tau)\varepsilon^i] \exp(\gamma_1(t-\tau)) + [\varepsilon[(r_1 + \alpha\gamma - \lambda_1)(-(\delta_1 T_{20}'(\tau) + P_1(T_{10}(\tau)) + \frac{C_1}{D_1}(\delta_2 T_{10}'(\tau) + P_2(T_{10}(\tau)))] + \sum_{i=2}^{n} F_{2i}(\tau)\varepsilon^i] \exp(\gamma_2(t-\tau))]\mathrm{d}\tau + O(\varepsilon^{n+1}),$$
$$0 < \varepsilon << 1 \quad (18)$$

$$T_2(t) = [C_2 \exp(r_1 t) + D_2 \exp(r_2 t) - \frac{1}{\sqrt{\Delta}} \int_0^t [[\varepsilon[\frac{D_1}{C_1}(r_2 + \alpha\gamma - \lambda_1)(\delta_1 T_{20}'(\tau) + P_1(T_{10}(\tau))) +$$

$$(r_1+\alpha\gamma-\lambda_1)\,(\delta_2 T_{10}'(\tau)+P_2(T_{10}(\tau)))]+$$

$$\sum_{i=2}^{n}F_{1i}(\tau)\varepsilon^i]\exp(\gamma_1(t-\tau))+[(r_2+\alpha\gamma-\lambda_1)$$

$$(-\frac{D_1}{C_1}(\lambda_1 T_{10}'(\tau)+P_1(T_{10}(\tau)))+(\lambda_2 T_{20}'(\tau)+$$

$$P_2(T_{20}(\tau))))]+\sum_{i=2}^{n}F_{2i}(\tau)\varepsilon^i]\exp(\gamma_2(t-\tau))]\mathrm{d}\tau,$$

$$0<\varepsilon<<1 \qquad (19)$$

From fixed point theorem, the equations (18) and (19) are valid expansions of the coupled system (1)−(2) for a sea-air oscillator model.

4 Example

Consider a special coupled system for a sea-air oscillator time delay model of interdecadal climate fluctuations. Let $P_1(T_1)=\sin T_1$, $P_2(T_2)=\cos T_2$. Thus the system (1)−(2) is

$$\frac{\mathrm{d}T_1}{\mathrm{d}t}=\alpha\gamma(T_2-T_1)-\delta_1 T_2(t-d_2)+\lambda_1 T_1-\varepsilon\sin T_1 \qquad (20)$$

$$\frac{\mathrm{d}T_2}{\mathrm{d}t}=\gamma(T_2-T_1)-\delta_2 T_1(t-d_1)+\lambda_2 T_2-\varepsilon\cos T_2 \qquad (21)$$

Let

$$T_j(t)=\sum_{i=0}^{\infty}T_{ji}(t)\varepsilon^i \qquad (22)$$

And developing the time delay functions $T_j(t-d_i)$ = $T_j(t-\varepsilon)$ in ε:

$$T_j(t-\varepsilon)=T_j(t)-\frac{\mathrm{d}T_j(t)}{\mathrm{d}t}\varepsilon+\frac{1}{2}\frac{\mathrm{d}^2T_j(t)}{\mathrm{d}t^2}\varepsilon^2+\cdots+$$

$$\frac{(-1)^k}{k!}\frac{\mathrm{d}^kT_j(t)}{\mathrm{d}t^k}\varepsilon^i+\cdots,\quad j=1,2 \qquad (23)$$

Substituting equations (22) and (23) into (20) and (21), developing nonlinear terms in ε, equating coefficients of the same powers of ε in both two sides for the equations respectively, for j = 0, 1, we obtain

$$\frac{\mathrm{d}T_{10}}{\mathrm{d}t}=(\lambda_1-\alpha\gamma)T_{10}+(\alpha\gamma-\delta_1)T_{20} \qquad (24)$$

$$\frac{\mathrm{d}T_{20}}{\mathrm{d}t}=-(\gamma+\delta_2)\,T_{01}+(\lambda_2+\gamma)T_{20} \qquad (25)$$

$$\frac{\mathrm{d}T_{11}}{\mathrm{d}t}=(\lambda_1-\alpha\gamma)T_{11}+(\alpha\gamma-\delta_1)T_{21}+\delta_1 T_{20}'+\sin T_{10} \qquad (26)$$

$$\frac{\mathrm{d}T_{21}}{\mathrm{d}t}=-(\gamma+\delta_2)T_{11}+(\lambda_2+\gamma)T_{21}+\delta_2 T_{10}'+\cos T_{20} \qquad (27)$$

It is easy to see that the general solution $T_{j0}(t)$ of the systems (24) and (25) is

$$T_{j0}(t)=C_j\exp(r_1 t)+D_j\exp(r_2 t),\quad j=1,2 \qquad (28)$$

We can obtain a set of solution T_{j1} (j =1, 2) for the systems (26) and (27) with zero initial conditions that

$$T_{11}(t)=-\frac{1}{\sqrt{\Delta}}\int_0^t[[(r_2+\alpha\gamma-\lambda_1)(\delta_1 T_{20}'(\tau)+\sin(\tau))-$$

$$\frac{D_1}{C_1}(r_1+\alpha\gamma-\lambda_1)(\delta_2 T_{10}'(\tau)+\cos(\tau))]$$

$$\exp(\gamma_1(t-\tau))+[(r_1+\alpha\gamma-\lambda_1)(-(\delta_1 T_{20}'(\tau)+$$

$$\sin(\tau))+\frac{C_1}{D_1}(\delta_2 T_{10}'(\tau)+\cos(\tau))]$$

$$\exp(\gamma_2(t-\tau))]\mathrm{d}\tau \qquad (29)$$

$$T_{21}(t)=-\frac{1}{\sqrt{\Delta}}\int_0^t[[\frac{D_1}{C_1}(r_2+\alpha\gamma-\lambda_1)\,(\delta_1 T_{20}'(\tau)+\sin(\tau))+$$

$$(r_1+\alpha\gamma-\lambda_1)\,(\delta_2 T_{10}'(\tau)+\cos(\tau))]$$

$$\exp(\gamma_1(t-\tau))+[(r_2+\alpha\gamma-\lambda_1)\,(-\frac{D_1}{C_1}(\lambda_1 T_{10}'(\tau)+$$

$$\sin(\tau))+(\lambda_2 T_{20}'(\tau)+\cos(\tau)))]$$

$$\exp(\gamma_2(t-\tau))]\mathrm{d}\tau \qquad (30)$$

Thus from equations (22) and (28)−(30), we have a set of first order approximation solution of the coupled system (20)–(21) for a sea-air oscillator model:

$$T_1(t)=C_1\exp(r_1 t)+D_1\exp(r_2 t)-\frac{\varepsilon}{\sqrt{\Delta}}$$

$$\int_0^t[[(r_2+\alpha\gamma-\lambda_1)(\delta_1 T_{20}'(\tau)+\sin(\tau))-$$

$$\frac{D_1}{C_1}(r_1+\alpha\gamma-\lambda_1)(\delta_2 T_{10}'(\tau)+\cos(\tau))]$$

$$\exp(\gamma_1(t-\tau))+[(r_1+\alpha\gamma-\lambda_1)(-(\delta_1 T_{20}'(\tau)+$$

$$\sin(\tau))+\frac{C_1}{D_1}(\delta_2 T_{10}'(\tau)+\cos(\tau)))]$$

$$\exp(\gamma_2(t-\tau))]\mathrm{d}\tau+O(\varepsilon^2),$$

$$-0<\varepsilon<<1 \qquad (31)$$

$$T_2(t) = C_2 \exp(r_1 t) + D_2 \exp(r_2 t) - \frac{\varepsilon}{\sqrt{\Delta}}$$

$$\int_0^t [[\frac{D_1}{C_1}(r_2 + \alpha\gamma - \lambda_1)(\delta_1 T_{20}'(\tau) + \sin(\tau)) +$$

$$(r_1 + \alpha\gamma - \lambda_1)(\delta_2 T_{10}'(\tau) + \cos(\tau))]$$

$$\exp(\gamma_1(t-\tau)) + [(r_2 + \alpha\gamma - \lambda_1)(-\frac{D_1}{C_1}(\lambda_1 T_{10}'(\tau) +$$

$$\sin(\tau)) + (\lambda_2 T_{20}'(\tau) + \cos(\tau)))]$$

$$\exp(\gamma_2(t-\tau))] \mathrm{d}\tau + O(\varepsilon^2),$$

$$0 < \varepsilon << 1 \qquad (32)$$

From fixed point theorem, the equations (31) and (32) are valid expansions of the coupled system (20)–(21) for a sea-air oscillator model.

5 Discussion

Atmospheric physics is a very complicated natural phenomenon. We consider the coupled system (1)−(2) for a sea-air oscillator time delay model of interdecadal climate fluctuations. In the coupled system, we deduced the extratropical temperature at approximate region from 25°N to 50°N (and 25°S to 50°S) (T_1) and the tropical temperature at approximate region from 20°S to 20°N (T_2). From the fraction of poleward atmospheric heat transport $\lambda_1 + \lambda_2 > (\alpha - 1)\gamma$ and $\alpha\gamma > \delta_1$, we know that the characteristic roots r_1 and r_2 possess different positive real parts. Thus the singular solution of the nonlinear system (1)−(2) is unstable. The path curves on the phase plane for the system (1)−(2) are away from the origin. Therefore, as the time (t) large enough, the solution of the system (1)−(2) may be take on the chaos T_{10}' behavior and it leads the equatorial Pacific to a never-ending warm state.

From the example, we can conclude that for small parameter ε, the asymptotic expansions are the good approximate expansions of the disturbed solution for the nonlinear model.

We obtained approximate solution by using the perturbation method. And we can obtain the prognoses of SST and related physical quantities.

6 Conclusions

We need to reduce basic models for the ENSO sea-air oscillator and solve them by using the approximate method. The perturbation method is a valid method.

The perturbed method is an approximate analytic method, which differs from general numerical method. The expansions of solution obtained through the perturbed method can be used analytically. We can further study the fixed quality and quantitative behaviors of the temperature anomaly in the equatorial eastern Pacific, the thermocline anomaly, the zonal wind stress anomaly and so on.

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