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Dissipative Travelling Wave Solution for El Niño Tropic Sea-air Coupled Oscillator

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Abstract: El Niño and Southern Oscillation (ENSO) is an interannual phenomenon involved in the tropical Pacific sea-air interactions. An asymptotic method of solving equations for the ENSO model is proposed. Based on a class of oscillator of ENSO model and by employing a simple and valid method of the variational iteration, the coupled system for a sea-air oscillator model of interdecadal climate fluctuations is studied. Firstly, by introducing a set of functionals and computing the variationals, the Lagrange multipliers are obtained. And then, the generalized variational iteration expressions are constructed. Finally, by selecting appropriate initial iteration, and from the iterations expressions, the approximations of solution for the sea-air oscillator ENSO model are solved successively. The approximate dissipative travelling wave solution of equations for corresponding ENSO model is studied. It is proved from the results that the method of the variational iteration can be used for analyzing the sea surface temperature anomaly in the equatorial Pacific of the sea-air oscillator for ENSO model.

Keywords: El Niño-Southern Oscillator model; variational iteration; sea-air oscillator

1 Introduction

El Niño and Southern Oscillation (ENSO) is an abnormal event happening in the atmosphere and tropical Pacific Ocean. The oscillatory nature of ENSO involves both positive and negative sea-air feedbacks. And a positive sea surface temperature (SST) anomaly in the equatorial eastern Pacific has been studied. This anomaly reduces the zonal SST gradient and the strength of the southern oscillation circulation, resulting in weaker trade winds around the equator. The weaker trade winds in turn cause the ocean circulation to change and then reinforce the SST anomaly. The phenomenon of the ENSO is a very attractive object of research in the international academic circles. Some researchers have studied a class of ENSO model, such as the interdecadal climate fluctuations for the exchanges between the tropics and extratropics (Gu and Philander, 1997), the Kelvin waves in the tropics (Lin and Ceng, 1999), the meridional wind forced low frequency disturbances in the tropical ocean (Lin *et al*., 1999), the interaction change of the structure of the ENSO mode (An and Wang, 2000), the decadal variations in the subtropical cells and equatiorial Pacific SST (Nonaka *et al*., Xie and McCreary, 2001), the slowdown of the meridional overturning circulation in the upper Pacific Ocean (McPhaden and Zhang, 2002), the stability for linear and non-linear evolution equations (Lin *et al*., 2002; Wang, 2002), and the perturbed solution of the coupled ocean-atmosphere model for ENSO (Lin and Mo, 2004). Mo *et al*. also studied the perturbed solution of a sea-air oscillator model for the ENSO (Mo *et al*., 2006a), the homotopic method of solving a class of ENSO sea-air oscillator (Mo *et al*., 2006b; Mo, 2009b), the mechanism

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of the equatorial eastern ENSO (Mo *et al*., 2006c), the sea-air oscillator model for the ENSO (Mo *et al*., 2007a), the perturbed mechanism of western boundary undercurrents in the Pacific (Mo *et al*., 2007b), the asymptotic solution for a class of sea-air oscillator model for El-Nino-southern oscillation (Mo and Lin, 2008a), and the travelling wave solution for ENSO tropic sea-air coupled oscillator (Mo and Lin, 2008b).

Recently, approximation methods have been developed and improved (Bobkova, 2005; Marques, 2006; Ni and Wei, 2006). Using the asymptotic method, Mo discussed a class of nonlinear problems (Mo, 2009a; 2009c; 2009d; 2009e). In this paper we study a class of ENSO model by using a simple and special method in approximate theory. And it is proved from the results that the method of the variational iteration can be used for analyzing the sea surface temperature anomaly in the equatorial Pacific of the sea-air oscillator for ENSO model.

2 Model of Sea-Air Oscillator of ENSO

In the ENSO oscillator model, the variations of both the eastern and western Pacific anomaly patterns should be considered. We study only an oscillator model, i.e. a sea-air coupled oscillator model. In this paper, by using the method of variational iteration (He, 2002; 2006), approximate solution of the equations for the model is obtained.

We now consider the following coupled equations, which describes the coupled effect between the atmosphere and ocean in tropical Pacific (Lin and Ceng, 1999):

$$\frac{\partial u^a}{\partial t}+\frac{\partial \phi}{\partial x}+Du^a=0,\quad \frac{\partial \phi}{\partial t}+C_a^2\frac{\partial u^a}{\partial x}+D\phi=-Q$$

$$\frac{\partial u^s}{\partial t}+\frac{\partial \eta}{\partial x}+du^s=\tau^x,\quad \frac{\partial \eta}{\partial t}+C_s^2\frac{\partial u^s}{\partial x}+d\eta=0$$

and also consider the coupled relations of the temperature functions of atmosphere and ocean:

$$\beta y u^a+\frac{\partial \phi}{\partial y}=0,\quad \beta y u^s+\frac{\partial \eta}{\partial y}=0$$

where u^a and u^s are the temperature functions of atmosphere and ocean respectively; ϕ is the gravitational potential function of atmosphere; η is the potential function of mixed layer thickness of ocean; C_a and C_s are the gravitational wave speeds of atmosphere and ocean respectively; β is the Rossby parameter; D and d are Rayleigh friction and Newton cooling coefficient of ocean and atmosphere respectively; Q is the calefaction rate of ocean for atmosphere; τ^x is the wind stress. We take $Q=K_OT$ and $\tau^x=K_Su^a$, where T is the sea surface temperature, K_O and K_S are rates of the sea surface temperature and the temperature functions of atmosphere respectively. And let $T=\kappa\eta$, where κ is a rate of the mixed layer thickness of ocean. Thus we have $Q=K_H\eta$ and $K_H=\kappa K_O$. Through the dimensioness treatment: $x=C(1/2\beta)x'$, $t=(1/2\beta)t'$, $y_a=(1/2\beta)^{1/2}y_a'$, $y_s=C_s(1/2\beta)^{1/2}y_s'$, $u^a=C_au^{a\prime}$, $u^s=C_au^{s\prime}$, $\phi=C_a^2\phi'$, $\eta=C_s^2\eta'$. The original coupled equations can be turned into the following coupled equations (form the prime is omitted):

$$\frac{\partial u^a}{\partial t}+\alpha^{-1}\frac{\partial \phi}{\partial x}+\tilde{D}u^a=0,\ \frac{\partial \phi}{\partial t}+\alpha^{-1}\frac{\partial u^a}{\partial x}+\tilde{D}\phi=-\tilde{K}_H\eta \tag{1}$$

$$\frac{\partial \eta}{\partial t}+\frac{\partial u^s}{\partial x}+\tilde{d}\eta=0,\quad \frac{\partial u^s}{\partial t}+\frac{\partial \eta}{\partial x}+\tilde{d}u^s=\tilde{K}_Su^a \tag{2}$$

with

$$\frac{1}{2}yu^a+\frac{\partial \phi}{\partial y}=0,\quad \frac{1}{2}yu^s+\frac{\partial \eta}{\partial y}=0 \tag{3}$$

where $a=C_S/C_a$, $\tilde{D}=D(2\beta)C_a$, $\tilde{d}=(C_S^2d)/(2\beta)$.

From equations (1) and (2), we have

$$\frac{\partial(u^a\pm\phi)}{\partial t}\pm\alpha^{-1}\frac{\partial(u^a\pm\phi)}{\partial x}+\tilde{D}(u^a\pm\phi)=\mp\tilde{K}_H\eta \tag{4}$$

$$\frac{\partial(u^s\pm\eta)}{\partial t}\pm\frac{\partial(u^s\pm\eta)}{\partial x}+\tilde{d}(u^s\pm\eta)=\pm\tilde{K}_Su^a \tag{5}$$

We first consider the calefaction rate ($Q=0$) of ocean for atmosphere and the wind stress ($\tau^x=0$), thus the corresponding non-dissipative equations for equations (3) and (4) are as follows:

$$\frac{\partial(u^a\pm\phi)}{\partial t}\pm\alpha^{-1}\frac{\partial(u^a\pm\phi)}{\partial x}+\tilde{D}(u^a\pm\phi)=0 \tag{6}$$

$$\frac{\partial(u^s\pm\eta)}{\partial t}\pm\frac{\partial(u^s\pm\eta)}{\partial x}+\tilde{d}(u^s\pm\eta)=0 \tag{7}$$

The characteristic equations of the non-dissipative equations (6) and (7) are

$$\frac{\mathrm{d}t}{1}=\frac{\mathrm{d}x}{\pm\alpha^{-1}}=\frac{\mathrm{d}(\tilde{u}^a\pm\tilde{\phi})}{-\tilde{D}(\tilde{u}^a\pm\tilde{\phi})},\quad \frac{\mathrm{d}t}{1}=\frac{\mathrm{d}x}{\pm1}=\frac{\mathrm{d}(\tilde{u}^s\pm\tilde{\eta})}{-\tilde{d}(\tilde{u}^s\pm\tilde{\eta})} \tag{8}$$

It is not difficult to obtain the first integrals of Equation (8), given as $x\mp\alpha^{-1}t=c_1$ and $\ln(\tilde{u}^a\pm\tilde{\phi})+\tilde{D}t=c_2$, $x\mp t=c_3$ and $\ln(\tilde{u}^a\pm\tilde{\eta})+\tilde{d}t=c_4$, where c_i (i = 1, 2, 3, 4) are arbitrary constants. Thus the general solutions for equations (4) and (5) are obtained as follows:

$$\tilde{u}^a \pm \tilde{\phi} = \exp(-\tilde{D}t + \Phi_{01}^{\pm}(x \mp \alpha^{-1}t, y)) \quad (9)$$

$$\tilde{u}^s \pm \tilde{\eta} = \exp(-\tilde{d}t + \Phi_{02}^{\pm}(x \mp t, y)) \quad (10)$$

where $\Phi_{0i}^{\pm}$ (i = 1, 2) are arbitrary functions. Considering Equation (3), we have

$$\Phi_{01}^{\pm}(x \mp \alpha^{-1}t, y) = \Psi_{01}^{\pm}(x \mp \alpha^{-1}t) \mp \frac{1}{4}y^2$$

$$\Phi_{02}^{\pm}(x \mp t, y) = \Psi_{02}^{\pm}(x \mp t) \mp \frac{1}{4}y^2$$

where $\Psi_{0i}^{\pm}$ (i = 1, 2, 3, 4) are arbitrary functions, too. And from equations (9) and (10), we obtain the following traveling wave solution:

$$\tilde{u}^a = \frac{1}{2}[\exp(-\tilde{D}t + \Psi_{01}^{-}(x - \alpha^{-1}t) - \frac{y^2}{4}) + \exp(-\tilde{D}t + \Psi_{01}^{+}(x + \alpha^{-1}t) + \frac{y^2}{4})] \quad (11)$$

$$\tilde{u}^s = \frac{1}{2}[\exp(-\tilde{d}t + \Psi_{02}^{-}(x - t) - \frac{y^2}{4}) + \exp(-\tilde{d}t + \Psi_{02}^{+}(x + t) + \frac{y^2}{4})] \quad (12)$$

$$\tilde{\phi} = \frac{1}{2}[\exp(-\tilde{D}t + \Psi_{01}^{-}(x - \alpha^{-1}t) - \frac{y^2}{4}) - \exp(-\tilde{D}t + \Psi_{0}^{+}(x + \alpha^{-1}t) + \frac{y^2}{4})] \quad (13)$$

$$\tilde{\eta} = \frac{1}{2}[\exp(-\tilde{d}t + \Psi_{02}^{-}(x - t) - \frac{y^2}{4}) - \exp(-\tilde{d}t + \Psi_{02}^{+}(x + t) + \frac{y^2}{4})] \quad (14)$$

3 Variational Iteration

Set $\tilde{v}_\pm = u^a \pm \phi, \tilde{w}_\pm = u^s \pm \eta$, and inducting the travelling wave transforms $\xi_\pm = x \pm \alpha^{-1}t$, $\zeta_\pm = x \mp t$. Then equations (6) and (7) are

$$\frac{\partial \tilde{v}_\pm}{\partial \xi_\pm} + \tilde{D}v_\pm = 0, \quad \frac{\partial \tilde{w}_\pm}{\partial \zeta} + \tilde{d}w_\pm = 0 \quad (15)$$

Introducing the following functionals $F_{1\pm}[v_\pm]$, $F_{i2}[w_\pm]$ (He, 2002; 2006):

$$F_{1\pm}[v_\pm] = v_\pm - \int_{-\infty}^{\xi_\pm} \lambda_{1\pm}(\xi_{1\pm})(\frac{\partial v_\pm}{\partial \xi_{1\pm}} + \tilde{D}v_\pm \pm \tilde{K}_H\overline{\eta})\,\mathrm{d}\xi_{1\pm} \quad (16)$$

$$F_{2\pm}[w_\pm] = w_\pm - \int_{-\infty}^{\zeta_\pm} \lambda_{2\pm}(\zeta_{1\pm})(\frac{\partial w_\pm}{\partial \zeta_{1\pm}} + \tilde{d}\, w_\times \mp \tilde{K}_S\overline{u}^a)\,\mathrm{d}\zeta_{1\pm} \quad (17)$$

where $\overline{v}_\pm$, $\overline{w}_\pm$, $\overline{\eta}_\pm$, $\overline{u}_\pm^s$ are the restricted variables of $v_\pm$, $w_\pm$, $\eta_\pm$, $u_\pm^s$, respectively (He, 2002) and $\lambda_{i\pm}$ (i=1, 2) are Lagrange multipliers. Computing the variationals $\delta F_{1\pm}[v_\pm]$, $\delta F_{2\pm}[w_\pm]$ of the functionals in equations (16) and (17), we have

$$\delta F_{1\pm}[v_\pm] = \delta v_\pm - \lambda_{1\pm}\big|_{\xi_{1\pm}=\xi_\pm}\delta v_\pm + \int_{-\infty}^{\xi_\pm}(\frac{\partial \lambda_{1\pm}}{\partial \xi_{1\pm}} + \tilde{D}\lambda_{1\pm})\delta v_\pm\,\mathrm{d}\xi_{1\pm} \quad (18)$$

$$\delta F_{2\pm}[w_\pm] = \delta w_\pm - \lambda_{2\pm}\big|_{\zeta_{1\pm}=\zeta_\pm}\delta w_\pm + \int_{-\infty}^{\zeta_\pm}(\frac{\partial \lambda_{2\pm}}{\partial \zeta_{1\pm}} + \tilde{d}\lambda_{2\pm})\delta w_\pm\,\mathrm{d}\zeta_{1\pm} \quad (19)$$

Then we have only

$$\frac{\partial \lambda_{1\pm}}{\partial \xi_{1\pm}} + \tilde{D}\lambda_{1\pm} = 0, \quad \frac{\partial \lambda_{2\pm}}{\partial \zeta_{1\pm}} + \tilde{d}\lambda_{2\pm} = 0$$

$$\lambda_{1\pm}\big|_{\xi_{1\pm}=\xi_\pm} = 1, \quad \lambda_{2\pm}\big|_{\zeta_{1\pm}=\zeta_\pm} = 1$$

Thus we can have

$$\lambda_{1\pm} = \exp(-\tilde{D}(\xi_{1\pm} - \xi_\pm))$$

$$\lambda_{2\pm} = \exp(-\tilde{d}(\zeta_{1\pm} - \zeta_\pm))$$

That is

$$\lambda_1 = \frac{1}{2}(\lambda_{1+} + \lambda_{1-}) = \frac{1}{2}[\exp(-\tilde{D}(\xi_{1+} - \xi_+)) + \exp(-\tilde{D}(\xi_{1-} - \xi_-))]$$

$$\lambda_2 = \frac{1}{2}(\lambda_{1+} - \lambda_{1-}) = \frac{1}{2}[\exp(-\tilde{D}(\xi_{1+} - \xi_+)) - \exp(-\tilde{D}(\xi_{1-} - \xi_-))]$$

$$\lambda_3 = \frac{1}{2}(\lambda_{2+} + \lambda_{2-}) = \frac{1}{2}[\exp(-\tilde{d}(\zeta_{1+} - \zeta_+)) + \exp(-\tilde{d}(\zeta_{1-} - \zeta_-))]$$

$$\lambda_4 = \frac{1}{2}(\lambda_{2+} - \lambda_{2-}) = \frac{1}{2}[\exp(-\tilde{d}(\zeta_{1+} - \zeta_+)) - \exp(-\tilde{d}(\zeta_{1-} - \zeta_-))]$$

From the above depictions, we construct the following iteration sequences $\{u_n^a\}$, $\{u_n^s\}$, $\{\phi_n\}$, $\{\eta_n\}$ for the solutions of the original coupled equations (1)–(3):

$$u_{n+1}^a = u_n^a - \frac{1}{2}\int_{-\infty}^{x}\int_0^t[\exp(-\tilde{D}((x_1 - x) + \alpha^{-1}(t_1 - t)) + \exp(-\tilde{D}((x_1 - x) - \alpha^{-1}(t_1 - t))] \times (\frac{\partial u_n^a}{\partial t_1} + \alpha^{-1}\frac{\partial \phi_n}{\partial x} + \tilde{D}\,u_n^a)\,\mathrm{d}t_1\,\mathrm{d}x_1, \ n = 0, 1, \cdots \quad (20)$$

$$u_{n+1}^{s}=u_{n}^{s}-\frac{1}{2}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{d}((x_1-x)+(t_1-t))+\exp(-\tilde{d}((x_1-x)-(t_1-t))]\times$$
$$(\frac{\partial u_n^s}{\partial t_1}+\frac{\partial \phi_n}{\partial x}+\tilde{d}\,u_n^{sa}-\tilde{K}_S u_n^a)\,\mathrm{d}t_1\,\mathrm{d}x_1,\ n=0,1,\cdots \tag{21}$$

$$\phi_{n+1}=\phi_n-\frac{1}{2}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{D}((x_1-x)+\alpha^{-1}(t_1-t))-\exp(-\tilde{D}((x_1-x)-\alpha^{-1}(t_1-t))]\times$$
$$(\frac{\partial \phi_n}{\partial t_1}+\alpha^{-1}\frac{\partial u_n^a}{\partial x}+\tilde{D}\,\phi_n+K_H\eta_n)\,\mathrm{d}t_1\,\mathrm{d}x_1,\ n=0,1,\cdots \tag{22}$$

$$\eta_{n+1}=\eta_n-\frac{1}{2}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{d}((x_1-x)+(t_1-t))-\exp(-\tilde{d}((x_1-x)-(t_1-t))]\times$$
$$(\frac{\partial \eta_n}{\partial t_1}+\frac{\partial u_n^s}{\partial x}+\tilde{d}\,\eta_n)\,\mathrm{d}t_1\,\mathrm{d}x_1,\ n=0,1,\cdots \tag{23}$$

We can know that if selecting initial approximations u_0^a, u_0^s, ϕ_0, η_0 are the solutions of the corresponding reduced equations for equations (1)–(3), then $\{u_n^a\}$, $\{u_n^s\}$, $\{\phi_n\}$, $\{\eta_n\}$ are the uniformly convergent sequence functions using the fixed point theorem (de Jager and Jiang, 1996). Therefore $(u^a=\lim_{n\to\infty}u_n^a, u^s=\lim_{n\to\infty}u_n^s$, $\phi=\lim_{n\to\infty}\phi_n$, $\eta=\lim_{n\to\infty}\eta_n)$ is a set of solution for the coupled equations between the atmosphere and ocean in tropical Pacific.

4 Approximate Solution of Coupled Equations

In order to obtain the approximate solution of coupled equations (1)–(3), we assume that the zero-th order approximate solutions are decided by Equation (15)

$$v_{0\pm}=u_0^a\pm\phi_0=\tilde{v}_\pm,\ w_{0\pm}=u_0^s\pm\eta_0=\tilde{w}_\pm$$

Therefore, the solutions of equations (11)–(14) are

$$u_0^a=\frac{1}{2}[\exp(-\tilde{D}t+\Psi_{01}^{-}(x-\alpha^{-1}t)-\frac{y^2}{4})+$$
$$\exp(-\tilde{D}t+\Psi_{01}^{+}(x+\alpha^{-1}t)+\frac{y^2}{4})] \tag{24}$$

$$u_0^s=\frac{1}{2}[\exp(-\tilde{d}t+\Psi_{02}^{-}(x-t)-\frac{y^2}{4})+$$
$$\exp(-\tilde{d}t+\Psi_{02}^{+}(x+t)+\frac{y^2}{4})] \tag{25}$$

$$\phi_0=\frac{1}{2}[\exp(-\tilde{D}t+\Psi_{01}^{-}(x-\alpha^{-1}t)-\frac{y^2}{4})-$$
$$\exp(-\tilde{D}t+\Psi_{0}^{+}(x+\alpha^{-1}t)+\frac{y^2}{4})] \tag{26}$$

$$\eta_0=\frac{1}{2}[\exp(-\tilde{d}t+\Psi_{02}^{-}(x-t)-\frac{y^2}{4})-$$
$$\exp(-\tilde{d}t+\Psi_{02}^{+}(x+t)+\frac{y^2}{4})] \tag{27}$$

From the iteration expansions of the relation (20)–(23), we obtain first order approximate solutions u_1^a, u_1^s, ϕ_1, η_1 for the equations (1)–(3).

$$u_1^a=\frac{1}{2}[\exp(-\tilde{D}t+\Psi_{01}^{-}(x-\alpha^{-1}t)-\frac{y^2}{4})+\exp(-\tilde{D}t+\Psi_{01}^{+}(x+\alpha^{-1}t)+\frac{y^2}{4})] \tag{28}$$

$$u_1^s=\frac{1}{2}[\exp(-\tilde{d}t+\Psi_{02}^{-}(x-t)-\frac{y^2}{4})+\exp(-\tilde{d}t+\Psi_{02}^{+}(x+t)+\frac{y^2}{4})]+$$
$$\frac{K_S}{4}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{d}((x_1-x)+\alpha^{-1}(t_1-t))+\exp(-\tilde{d}((x_1-x)-\alpha^{-1}(t_1-t))]\times$$
$$[\exp(-\tilde{D}t_1+\Psi_{01}^{-}(x_1-\alpha^{-1}t_1)-\frac{y^2}{4})+\exp(-\tilde{D}t_1+\Psi_{01}^{+}(x_1+\alpha^{-1}t_1)+\frac{y^2}{4})]\,\mathrm{d}t_1\,\mathrm{d}x_1 \tag{29}$$

$$\phi_1=\frac{1}{2}[\exp(-\tilde{D}t+\Psi_{01}^{-}(x-\alpha^{-1}t)-\frac{y^2}{4})-\exp(-\tilde{D}t+\Psi_{0}^{+}(x+\alpha^{-1}t)+\frac{y^2}{4})]-$$
$$\frac{K_H}{4}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{D}((x_1-x)+\alpha^{-1}(t_1-t))-\exp(-\tilde{D}((x_1-x)-\alpha^{-1}(t_1-t))]\times$$
$$[\exp(-\tilde{d}t_1+\Psi_{02}^{-}(x_1-t_1)-\frac{y^2}{4})-\exp(-\tilde{d}t_1+\Psi_{02}^{+}(x_1+t_1)+\frac{y^2}{4})]\mathrm{d}t_1\mathrm{d}x_1 \tag{30}$$

$$\eta_1 = \frac{1}{2}[\exp(-\tilde{d}t + \Psi_{02}^-(x-t) - \frac{y^2}{4}) - \exp(-\tilde{d}t + \Psi_{02}^+(x+t) + \frac{y^2}{4})] \tag{31}$$

From the iteration expansions of the relation (20)–(23) and equations (26)–(29), we obtain second order approximate solutions u_2^a, u_2^s, ϕ_2, η_2 for the equations (1)–(3):

$$u_2^a = \frac{1}{2}[\exp(-\tilde{D}t + \Psi_{01}^-(x-\alpha^{-1}t) - \frac{y^2}{4}) + \exp(-\tilde{D}t + \Psi_{01}^+(x+\alpha^{-1}t) + \frac{y^2}{4})]$$

$$\begin{aligned} u_2^s = {} & \frac{1}{2}[\exp(-\tilde{d}t + \Psi_{02}^-(x-t) - \frac{y^2}{4}) + \exp(-\tilde{d}t + \Psi_{02}^+(x+t) + \frac{y^2}{4})] + \\ & \frac{K_S}{4}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{d}((x_1-x)+\alpha^{-1}(t_1-t)) + \exp(-\tilde{d}((x_1-x)-\alpha^{-1}(t_1-t))] \times \\ & [\exp(-\tilde{D}t_1 + \Psi_{01}^-(x_1-\alpha^{-1}t_1) - \frac{y^2}{4}) + \exp(-\tilde{D}t_1 + \Psi_{01}^+(x_1+\alpha^{-1}t_1) + \frac{y^2}{4})]\,\mathrm{d}t_1\,\mathrm{d}x_1 - \\ & \frac{1}{2}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{d}((x_1-x)+(t_1-t)) + \exp(-\tilde{d}((x_1-x)-(t_1-t))] \times \\ & (\frac{\partial u_1^s}{\partial t_1} + \frac{\partial \phi_1}{\partial x} + \tilde{d}\,u_1^a - \tilde{K}_S u_1^a)\,\mathrm{d}t_1\,\mathrm{d}x_1 \end{aligned}$$

$$\begin{aligned} \phi_2 = {} & \frac{1}{2}[\exp(-\tilde{D}t + \Psi_{01}^-(x-\alpha^{-1}t) - \frac{y^2}{4}) - \exp(-\tilde{D}t + \Psi_{0}^+(x+\alpha^{-1}t) + \frac{y^2}{4})] - \\ & \frac{K_H}{4}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{D}((x_1-x)+\alpha^{-1}(t_1-t)) - \exp(-\tilde{D}((x_1-x)-\alpha^{-1}(t_1-t))] \times \\ & [\exp(-\tilde{d}t_1 + \Psi_{02}^-(x_1-t_1) - \frac{y^2}{4}) - \exp(-\tilde{d}t_1 + \Psi_{02}^+(x_1+t_1) + \frac{y^2}{4})]\mathrm{d}t_1\mathrm{d}x_1 - \\ & \frac{1}{2}\int_{-\infty}^{x}\int_{0}^{t}[\exp(-\tilde{D}((x_1-x)+\alpha^{-1}(t_1-t)) - \exp(-\tilde{D}((x_1-x)-\alpha^{-1}(t_1-t))] \times \\ & (\frac{\partial \phi_1}{\partial t_1} + \alpha^{-1}\frac{\partial u_1^a}{\partial x} + \tilde{D}\,\phi_1 + K_H\eta_1)\,\mathrm{d}t_1\,\mathrm{d}x_1 \end{aligned}$$

$$\eta_2 = \frac{1}{2}[\exp(-\tilde{d}t + \Psi_{02}^-(x-t) - \frac{y^2}{4}) - \exp(-\tilde{d}t + \Psi_{02}^+(x+t) + \frac{y^2}{4})]$$

where u_1^a, u_1^s, ϕ_1, η_1 denote by equations (26)–(29), and the arbitrary functions depend on Equation (3) and initial conditions for the original mode.

Analogously, using the iteration expansions of the relation (20)–(23) and equations (31)–(35), we can obtain m-th order approximate solutions u_m^a, u_m^s, ϕ_m, η_m $(m = 2, 3, \cdots)$ for the equations (1)–(3) of the coupled effect between the atmosphere and ocean in tropical Pacific, successively.

5 Discussion and Conclusions

In order to investigate the ENSO that is a very complicated natural phenomenon in atmospheric physics we need to develop a basic model for the sea-air oscillator. And we solve the ENSO model by using the approximation method. The method of the variational iteration is a simple and valid method.

In the method of the variational iteration, we first select appropriated variational Lagrange multipliers λ_i $(i = 1, 2, 3, 4)$ and assume that the non-dissipative equations are the zero-th order approximate traveling solution. Naturally, in this way, we can obtain faster the travelling wave solution of the coupled equations.

The method of the variational iteration is an approximate analytic method, which differs from general numerical method. The expansion of solution using the method of the variational iteration can be made by continuously performing analytic operations. Thus, from approximate solutions, we can study further the qualitative and quantitative behaviors of the temperature anomaly in the equatorial Pacific, the thermocline anomaly and the zonal wind stress anomaly and so on, about which no discussion is given in this paper.

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