Chin. Geogra. Sci. 2009 19(3) 203–210
DOI: 10.1007/s11769-009-0203-0
www.springerlink.com

Cassini Growth of Population Between Two Metropolitan Cities —A Case Study of Beijing-Tianjin Region, China

ZONG Yueguang, YANG Wei, MA Qiang, XUE Song
(*School of Geographic and Oceanographic Sciences, Nanjing University, Nanjing 210093, China*)

Abstract: The existing models of population distribution often focus on the region with a single city or even multiple centers, and lack the detailed explorations of the common and special type of urbanization areas with two centers. Taking Beijing-Tianjin region of China, which is a distinct dual-nuclei metropolitan area in the world, as an example and choosing Landsat-5 TM image in 2005, population, etc. as the data, this paper devotes to comprehending and illustrating a model of Cassini growth of population between the two metropolitan cities through the research of spatial population distribution pattern, aided with RS and GIS techniques. Main technical processes include Kriging interpolation of the population data and character simulation of the Cassini ovals. According to the calculation of *a/b*, a key characteristic index of Cassini growth model, the spatial structures of population distribution were given. When $a/b<1$, it is a curve with two separated loops with a population density more than 3000 persons/km^2. When $a/b=1$, it is a lemniscate curve with a population density about 3000 persons/km^2. When $1<a/b<\sqrt{2}$, it is a dog-bone shaped concave curve with a population density between 500–3000 persons/km^2. When $a/b=\sqrt{2}$, it is an oblate curve with a population density about 500 persons/km^2. When $a/b>\sqrt{2}$, there is an oval-shaped convex curve with a population density less than 500 persons/km^2. The results show that owing to the combined action and influence of the regional dual-nuclei, the population distribution of Beijing-Tianjin region is in accord with Cassini model significantly. Therefore, there is Cassini growth of population between the two metropolitan cities in Beijing-Tianjin region. In addition, the process of Cassini growth has extraordinarily instructive significance for judging the development stages of the dual-nuclei metropolitan areas.

Keywords: metropolitan city; dual-nuclei metropolitan area; urban spatial growth; Cassini growth model; Beijing-Tianjin region; China

1 Introduction

In the course of urbanization, the process of concentration and migration of population contains many fields of population, space, society, economy, etc. Therefore, population growth in urbanized areas is one of the key urbanization contents. The spatial growth of urban population attracts many researchers' attention because it is a remarkable phenomenon of landscape processes. There are many significant changes of landscape patterns in the high population density regions, such as metropolis in the world. In the last 20 years, geographers and planners have begun to think about the growth and form of cities with some new methods and ways. Mathematical and computer models have started to flourish (Makse et al., 1995; Yeh and Li, 1997; Clarke and Gaydos, 1998; Herold et al., 2003; Cheng and Masser, 2003; Lu et al., 2004; Hu and Lo, 2007; He et al., 2008).

Many researches focused on a single city growth but seldom worked on dual-nuclei metropolitan area. Clark (1951) brought forward the Negative Exponential Function Model of spatial population distribution in metropolitan areas in the 1950s, which became the classical model for the demonstration studies of urban spatial structure. Derived from it, some modern population distribution models, such as Cubic Spline Model, Switching Regression Model and Varying Coefficient Model,

Received date: 2009-01-12; accepted date: 2009-06-08
Foundation item: Under the auspices of National High-Tech Research and Development Program of China (863 Program) (No. 2007AA12Z235), National Natural Science Foundation of China (No. 40471058)
Corresponding author: ZONG Yueguang. E-mail: zongyg@126.com

were put forward (McDonald, 1989). Lu et al. (2002; 2003) brought forward the theory of Dual-nuclei Structure and made a certain mathematical derivation according to regional accessibility, but did not involve in the aspect of spatial population distribution influenced by dual-nuclei. Remotely sensed data were often used to get a general view of urban areas but were seldom integrated in a landscape modeling (Weber and Puissant, 2003). Weber and Hirsch (1997) proposed an integration of potential model results and GIS to assess development trends of urban settlements.

Taking Beijing-Tianjin region of China as an example, this paper devotes to comprehending and illustrating a model of Cassini growth of population between two metropolitan cities through the research of spatial population distribution pattern of dual-nuclei metropolitan area, aided with RS and GIS technology, to make a new exploration for the spatial population distribution and regional spatial structure within this special region.

2 Study Area, Data and Methods

2.1 Study area and data sources

Beijing-Tianjin-Hebei region is one of the three most important metropolitan regions along with Yangtze (Changjiang) River Delta region and Pearl (Zhujiang) River Delta region in China. The Beijing-Tianjin region in this paper is located in North China, including Beijing City, Tianjin City and Langfang City of Hebei Province. The distance between central areas of Beijing and Tianjin is about 140km. The former is the capital, and the political and cultural center of China, with a population of about 15×10^6 in 2005. The latter is the third biggest city in Chinese mainland, the gate of Beijing even of North China and Northwest China, with a population of about 10×10^6 in 2005. Langfang is located nearby the breaking point where attraction of two ends is equal, for about 70km between Beijing and Tianjin, with a population of about 500,000 in 2005. The study area covers an area of about 350,000km^2.

In this research, we collected a Landsat-5 TM image in 2005 (spatial resolution is 30m×30m) and 1∶100,000 topographic map of Beijing-Tianjin region in the 1980s as basic spatial data. Meanwhile, we have also collected population data of about 30 cities or towns within this area as basic attribute data (National Bureau of Statistics of China, 2006).

2.2 Methods

2.2.1 Urban spatial growth theory and Cassini growth model

With the emergence of Endogenous Growth Theory in the 1980s, economists have provided active but limited evidences on the existence and importance of urban spillover population (Malczewski, 2004). Jaffe et al. (1993) found that nearby inventors had a much higher incline to citing each others' patents, suggesting that the knowledge spillover was indeed localized. Carlino et al. (2007) explicitly examined the effects of employment density (jobs per square mile), city size (total employment), and others related to the rate of renovation across metropolitan areas in the United States. Loibl and Toetzer (2003) presented a simulated model for the past and future transformation of suburban land use patterns in the Vienna Region. Turok and Bailey (2004) examined the relationships between the competitiveness and collaboration in two significant cities within the UK. Ghordaf et al. (2004) provided a mathematical study of a urban dynamic model, by adjusting the ecological model proposed by Lotka and Volterra. Courtney et al. (2006) used the spatial economic data attained from four small English towns to measure the strength of economic integration between towns and their hinterlands and to estimate the magnitude of town-hinterland spillover effects.

Generally, the basic theories of urban spatial growth and urban spatial structure are considered to originate from Christaller's Central Place Theory and Perroux's Growth Pole Theory (Christaller, 1933; Perroux, 1950), and Lu's Pole-axis Theory (Lu, 1986; 1995), and Lu's Dual-nuclei Theory (Lu, 2002; 2003), both of which are based on the former two as well. The Cassini growth model cited in this paper describes the mode of a moving point moving around two attractive centers based on the different rules. The curve was first investigated by Cassini in 1680 when he was studying the relative motions of the Earth and the Sun. Cassini believed that the Sun traveled around the Earth relatively as one focus of an oval, with the Earth as another focus of the oval (Gray, 1997). This phenomenon might occur between two metropolitans when population increase around the centers.

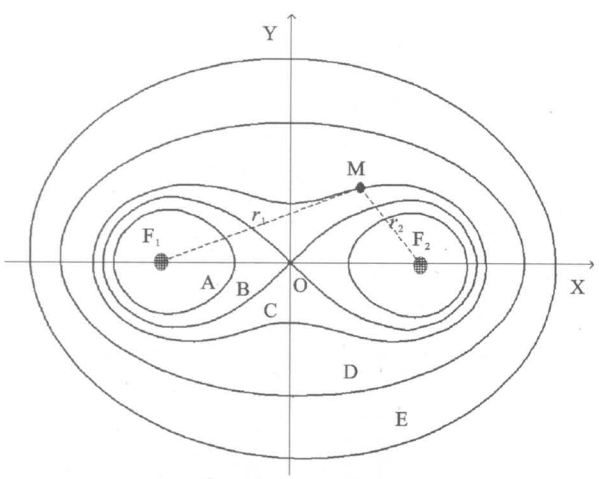
2.2.2 Application of potential model

The Cassini ovals are a family of quartic curves, also called Cassini ellipses. It described a collection of points whose products of the distances from the two fixed

points with a distance of $2b$ are a constant a^2, and it is defined as the two-center bipolar coordinates. Assuming that F_1, F_2 are two fixed points and r_1, r_2 are the distances from a certain point (M) of the curves to the two fixed points respectively and θ is the polar angle, if $\overline{F_1F_2}=2b$ and $r_1r_2=a^2$, taking the linear of $\overline{F_1F_2}$ as a polar axis and the middle point O as a pole, then the polar equation of the ovals can be deduced as Equation (1) (Gray, 1997):

$$r^2=b^2\cos 2\theta \pm \sqrt{b^4\cos^2 2\theta-(b^4-a^4)} \quad (1)$$

The shape of the curve depends on a/b. If $a/b<1$, the curve consists of two loops (A). If $a/b=1$, the curve shows a lemniscate, namely the *Bernoulli Lemniscate* (B). If $a/b>1$: when $1<a/b<\sqrt{2}$, the curve is concave single loop with a dog-bone shape (C); when $a/b\geq\sqrt{2}$, the curve is convex single loop with an oval (D, E). Cassini ovals are anallagmatic curves (Fig. 1).



X, Y are the abscissa and ordinate of the polar coordinates and F_1, F_2 are the two fixed points of bipolar respectively; M is a certain point of the curves of the Cassini ovals; A is the curve with two loops; B is the *Bernoulli Lemniscate*; C is the concave single loop with a dog-bone shape; D and E are convex single loops with an oval

Fig. 1 Normal mathematical description of Cassini ovals

2.2.3 Method and main steps

In this research, we applyed an ordinary Kriging interpolation method with the population data to finding a suitable growth model in this area. Kriging interpolation developed from Matheron and Krige is a theory of Regionalized Variables. Its basic premise is that every unknown point can be established by the weighed sum of the known points. Kriging provides the exact interpolation for it minimizes the error of predicted values which are estimated by spatial distribution of the predicted values. The ordinary Kriging models integrated in the ArcGIS software can be described as Equation (2) (Davis, 2002):

$$Z(x_0)=\sum_{i=1}^{n} w_iZ(x_i);\ \sum_{i=1}^{n} w_i=1 \quad (2)$$

where $Z(x_i)$ is the measured value at the ith location (i=1, 2, 3, …, n), n is the number of the locations, $Z(x_0)$ is predicted value, and w_i is the Kriging weight. The establishment of weights is under the consideration of not only the distance between the measured points and the predicted ones, but also the overall spatial arrangement among the measured points. Main steps are as follows.

Step 1: Make coordinate registration and geometric correction in order to generate a digital topographic map with a high precision, using the 1∶100,000 topographic map and assigned corresponding spatial reference system.

Step 2: Extract the subject information of built-up area and traffic, using visual interpretation method (Figs. 2 and 3).

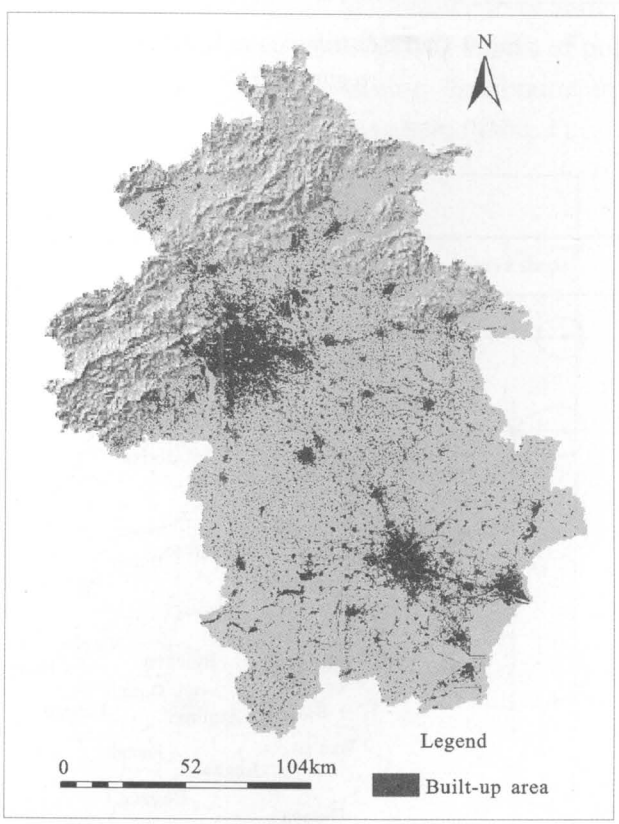


Fig. 2 Built-up area distribution of Beijing-Tianjin region in 2005

Step 3: Scan the topographic map into digital image, after matching and correcting, to digitize the border of the administration and form the closed polygons (Fig. 4).

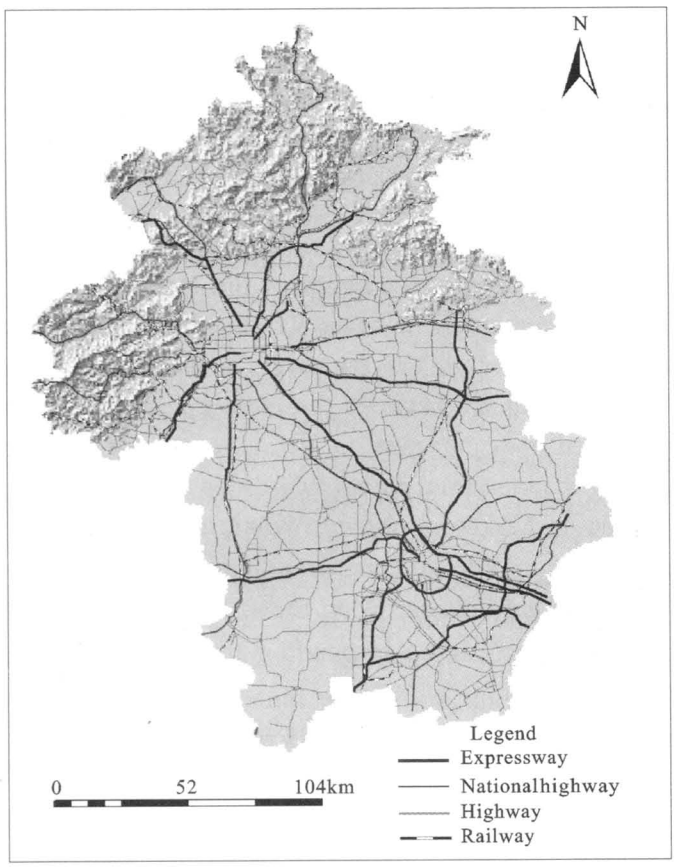


Fig. 3 Traffic distribution of Beijing-Tianjin region in 2005

Fig. 4 Administrative division of Beijing-Tianjin region in 2005

Step 4: Valuate corresponding attributes, establish topological relation for those polygons, and build spatial attribute database, according to the data of urban population.

Step 5: Apply Kriging interpolation with the population data to delineating the Cassini ovals according to the population distribution.

3 Results and Analysis

3.1 Pattern description

The Kriging interpolation result of population density in 2005 is shown in Fig. 5. Taking this as a basis, the Cassini ovals can be delineated out approximately as shown in Fig. 6.

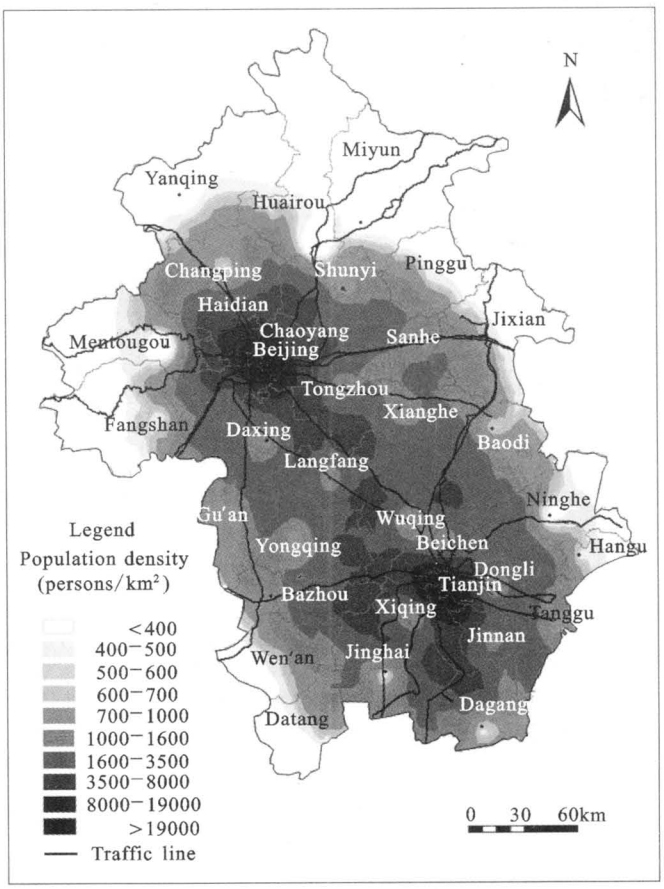


Fig. 5 Kriging interpolation of person density of Beijing-Tianjin region in 2005

3.2 Pattern types

The general trend of population distribution in Beijing-Tianjin region is indicated clearly in Fig. 5. In the central areas of Beijing and Tianjin, high-density population is aggregated, and radiated outwards non-uniformly along the main traffic arteries respectively, and de-

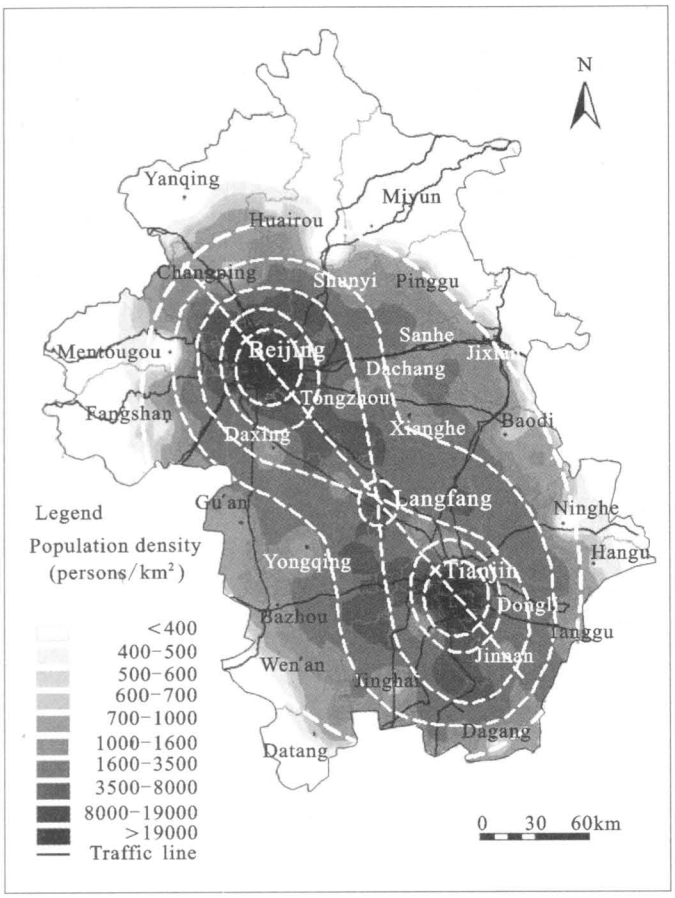


Fig. 6 Delineation of Cassini ovals of population density distribution in Beijing-Tianjin region in 2005

clines from centers to surrounding areas. A corridor with rather dense population appears between the two centers of high-density population. Further analysis indicates that the equipotent lines in the gravitational field of population gradient compose the Cassini ovals approximately. For the study area shows an oval-shape, dual-nuclei with high population density come into being in the urban central areas of Beijing and Tianjin and have a circle-layer-like degression according to the distance.

According to the calculation of a/b, the spatial structures of population distribution were given. When $a/b<1$, it is the curve with two separated loops with a population density more than 3000 persons/km^2. When $a/b=1$, it is a lemniscate curve with a population density about 3000 persons/km^2. When $1<a/b<\sqrt{2}$, it is a dog-bone shaped concave curve with a population density between 500–3000 persons/km^2. When $a/b=\sqrt{2}$, it is an oblate curve with a population density about 500 persons/km^2. When $a/b>\sqrt{2}$, there is an oval-shaped convex curve with a population density less than 500 persons/km^2. Thus, based on the five cases of Cassini curves above, we extract the corresponding characteristic values of population density. The spatial population distribution in this area could be attained in Cassini pattern (Table 1).

Table 1 Cassini pattern of population distribution in Beijing-Tianjin region in 2005

Population density (persons/km^2)	Theoretical formula	a	b	a/b	Curve shape
>3000	$a<b$	$0<a<70$	70	<1	
3000	$a=b$	70	70	=1	
500–3000	$b<a<b\sqrt{2}$	$70<a<98.96$	70	$1-\sqrt{2}$	
500	$a=b\sqrt{2}$	98.96	70	$=\sqrt{2}$	
<500	$a>b\sqrt{2}$	>98.96	70	$>\sqrt{2}$	

3.3 Model fitting

Here, a/b is a key characteristic index. Setting a/b is the I index. If the X axis is the I index and Y axis is the population density, then, we simulate a power-law mo-

del, which is similar to Clark's model, with a power around –3 in Equation (3) and Fig. 7.

$$y = 1840.9x^{-3.1069} \quad (3)$$

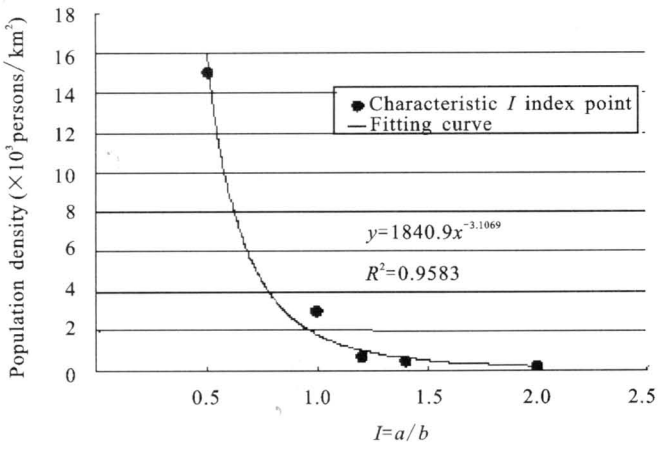


Fig. 7 Simulation curve between population density and *I* index

The model shows the Cassini pattern of spatial population distribution in dual-nuclei metropolitan region. The population presents high density distribution around the dual-nuclei and the areas enclosed by the lemniscate are the core population regions. Population density decays rapidly within the core areas, while reduces slowly beyond the core areas with the increase of distance. The population density of the breakpoint location is a significant parameter in the model.

3.4 Pattern evolvement analysis

Research shows that Cassini growth exists between the two metropolitan cities and this growth is related to the distribution of population density (Fig. 8). We describe the Cassini growth process as follows:

(1) The initial growing process. Two urban growth poles had a concentrate growth process for their lower population densities and economic developments. There

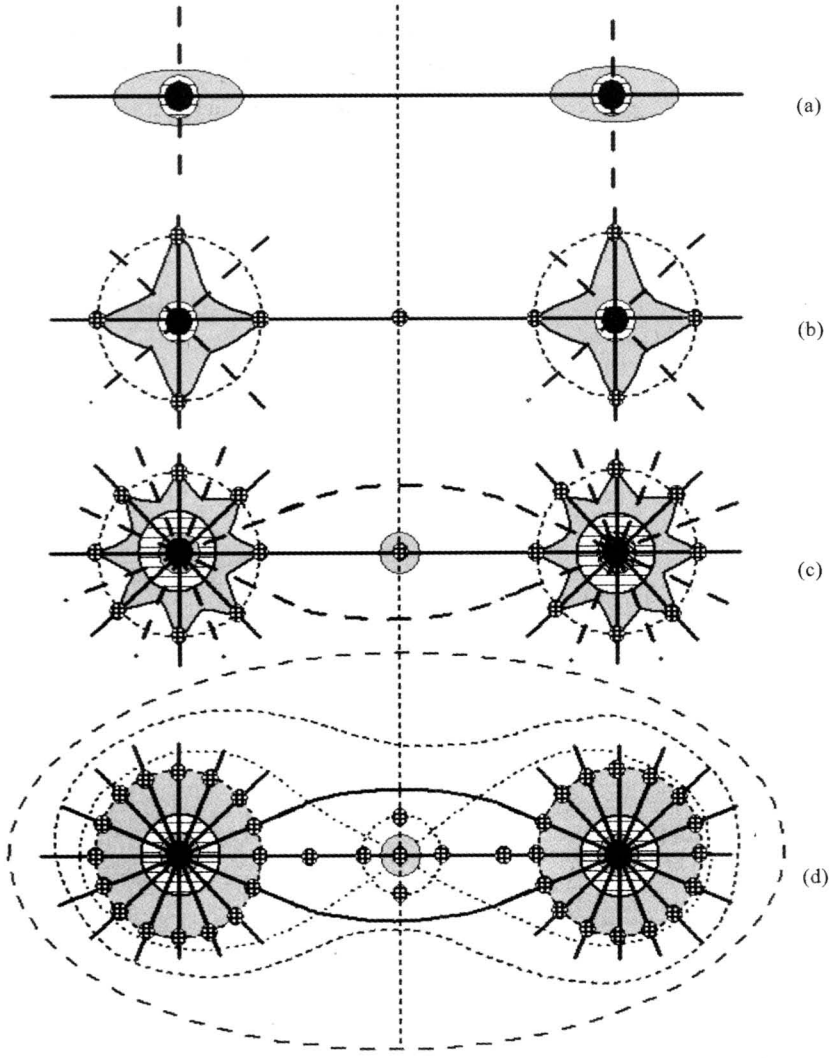


Fig. 8 Cassini growth processes between two metropolitan cities

was only main transportation corridor between two cities (Fig. 8a).

(2) The developing growing process. Two cities polarize and increase furthermore while the contact between them boosts up (Fig. 8b).

(3) The united growing process. Two metropolitan cities developed to form an obvious potential field. While the breakpoint converted towards correlative joint, a strong flow field was formed (Fig. 8c).

(4) The symbiosis process. The potential field of two metropolitan cities expanded toward both each other and neighboring areas. Pole-axis and corridor developed well, and a correlative joint was formed in the middle location. The whole dual-nuclei area combined closely and evolved together to become a region with a shape of Cassini oval (Fig. 8d).

4 Conclusions and Discussion

Compared with single or multiple centers regions, the dual-nuclei metropolitan area has some unique features on regional spatial structure and regional spatial growth. As a case study, the equipotent lines in the gravitational field of population gradient compose the Cassini ovals approximately, for the object region shows an oval-shape, dual-nuclei with high population density come into being in the central urban areas of Beijing and Tianjin and have a circle-layer-like degression according to distance. The corresponding characteristic values of population density shows that the population distribution of Beijing-Tianjin region in China accords with Cassini model significantly, owing to the combined action and influence of the regional dual-nuclei. As a result, there is Cassini growth of population between the two metropolitan cities. The power-law model which is simulated with the key characteristic index of *a/b* indicates that the spatial population distribution in dual-nuclei metropolitan region not only decays according to the increase of distance, but also has an extremely especial relation with the distance between dual-nuclei.

The Cassini growth model puts light on the product of two distances of each point to dual-nuclei. So it is a comprehensive consideration of the dual-nuclei potential power mechanism. Combining with five kinds of characteristic curves of the model, the process of Cassini growth of population has special instructional significance for judging the development stage of the dual-nuclei metropolitan areas, especially for judging the forming stage of breakpoint joints and the advanced stage of regional dual-nuclei integrating symbiosis growth. Of course, some factors such as the scale and capacity difference of dual-nuclei, artificial interferences in corridor or conjunction joint node of dual-nuclei will bring certain deviation and variation to the model. Those factors would be considered and the model would be improved in our future work.

References

Carlino G A, Chatterjee S, Hunt R M, 2007. Urban density and the rate of invention. *Journal of Urban Economics*, 61: 389–419. DOI: 10.1016/j.jue.2006.08.003

Cheng J Q, Masser I, 2003. Urban growth pattern modeling: A case study of Wuhan City, P. R. China. *Landscape and Urban Planning*, 62: 199–217. DOI: 10.1016/j.jue.2006.08.003

Christaller W, 1933. *Die Zentralen Orte in Sueddeutschland*. Jena: Gustau Fischer.

Clark C, 1951. Urban population densities. *Journal of Royal Statistics Society* (*Series A*), 114: 490–494.

Clarke K C, Gaydos L J, 1998. Loose-coupling a cellular automaton model and GIS: Long-term urban growth prediction for San Francisco and Washington/Baltimore. *International Journal of Geographical Information Science*, 12(7): 699–714.

Courtney P, Hillb G, Roberts D, 2006. The role of natural heritage in rural development: An analysis of economic linkages in Scotland. *Journal of Rural Studies*, 22: 469–484. DOI: 10.1016/j.jrurstud.2006.02.003

Davis J C, 2002. *Statistics and Data Analysis in Geology* (*3rd edition*). New York: John Wiley & Sons, Inc.

Ghordaf J E, Hbid M L, Arino O, 2004. A mathematical study of a two-regional population growth model. *C. R. Biologies*, 327: 977–982. DOI: 10.1016/j.crvi.2004.09.006

Gray A, 1997. *Modern Differential Geometry of Curves and Surfaces with Mathematic* (*2nd edition*). Boca Raton, FL: CRC Press.

Hanson J, 2001. Cassini: His ovals and egocentricity. *The Biblical Astronomer*, 11(96): 37–44.

He C Y, Okada N, Zhang Q F et al., 2008. Modelling dynamic urban expansion processes incorporating a potential model with cellular automata. *Landscape and Urban Planning*, 86: 79–91. DOI: 10.1016/j.landurbplan.2007.12.010

Herold M, Goldstein N C, Clarke K C, 2003. The spatiotemporal form of urban growth: Measurement, analysis and modeling. *Remote Sensing of Environment*, 86: 286–302. DOI: 10.1016/S0034-4257(03)00075-0

Hu Z Y, Lo C P, 2007. Modeling urban growth in Atlanta using logistic regression. *Computers, Environment and Urban Systems*, 31(6): 667–688. DOI: 10.1016/j.compenvurbsys.2006.11.001

Jaffe A B, Trajtenberg M, Henderson R, 1993. Geographic local-

ization of knowledge spillovers as evidenced by patent citations. *The Quarterly Journal of Economics*, 108(3): 577–598.

Loibl W, Toetzer T, 2003. Modeling growth and densification processes in suburban regions: Simulation of landscape transition with spatial agents. *Environmental Modelling & Software*, 18: 553–563. DOI: 10.1016/S1364-8152(03)00030-6

Lu Dadao, 1986. A scientific configuration of industrial allocation in China in 2000. *Scientia Geographica Sinica*, 6(2): 110–118. (in Chinese)

Lu Dadao, 1995. *Regional Development and the Spatial Structure*. Beijing: Science Press. (in Chinese)

Lu Yuqi, 2002. The mechanism of the model of dual-nuclei structure. *Acta Geographica Sinica*, 57(1): 85–95. (in Chinese)

Lu Yuqi, 2003. The mathematical derivation of the model of regional dual-nuclei structure. *Acta Geographica Sinica*, 58(3): 406–414. (in Chinese)

Lu Yuqi, Dong Ping, Yu Yongjun, 2004. Formative mechanism and regional effect of the twin cities of Shanghai and Suzhou. *Scientia Geographica Sinica*, 24(3): 305–313. (in Chinese)

Makse H A, Havlin S, Stanley H E, 1995. Modelling urban growth patterns. *Nature*, 377(6550): 608–612.

Malczewski J, 2004. GIS-based land-use suitability analysis: A critical overview. *Progress in Planning*, 62: 3–65. DOI: 10.1016/j.progress.2003.09.002

McDonald J, 1989. Econometric studies of urban population density: A survey. *Journal of Urban Economics*, 26(3): 361–385.

National Bureau of Statistics of China, 2006. China City Statistical Yearbook. Beijing: China Statistics Press. (in Chinese)

Perroux F, 1950. Economic space: Theory and application. The *Quarterly Journal of Economics*, 64: 89–104.

Turok I, Bailey N, 2004. Twin track cities? Competitiveness and cohesion in Glasgow and Edinburgh. *Progress in Planning*, 62: 135–204. DOI: 10.1016/j.progress.2004.04.001

Weber C, Hirsch J, 1997. Processus de croissance et limites urbaines. *Photo Interpretation*, 1–2: 21–33.

Weber C, Puissant A, 2003. Urbanization pressure and modeling of urban growth: Example of the Tunis Metropolitan Area. *Remote Sensing of Environment*, 86: 341–352. DOI: 10.1016/S0034-4257(03)00077-4

Yeh A G, Li X, 1997. An integrated remote sensing and GIS approach in the monitoring and evaluation of rapid urban growth for sustainable development in the Pearl River Delta, China. *International Planning Studies*, 2(2): 193–210. DOI: 10.1080/13563479708721678