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INFLUENCE OF SERIES OF SQUARE GRIDS ON FRACTAL DIMENSIONS
—A Case Study of Mountains of China′s Mainland

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ABSTRACT: MANDELBROT enunciated the uncertainty of the length of a coastline in his paper "How long is the coastline of Britain?" published in *Science* in 1967. The fractal concept was presented for the first time in that paper and has been applied to many fields ever since. Although fractal dimensions of lots of phenomena were calculated by the box-counting method, the quantitative influence of series of square grids on them is ignored. The issue is systematically discussed as a case study of the mountains of China′s Mainland in this paper. And some significant conclusions are drawn as follows: 1) Although the fractal character objectively exists in the mountains of China′s Mainland, and it does not vary with the changes of series of square grids, the fractal dimensions of the mountains of China′s Mainland are different with these changes. 2) The fractal dimensions of the mountains of China′s Mainland vary with the average lengths of sides of series of square grids. The fractal dimension of the mountains of China′s Mainland is the function of the average length of side of square grid. They conform to the formula $D=f(r)$ (where D is the fractal dimension, and r is the average length of side of square grid). 3) Different dots of data collection can affect the fractal dimension of the mountains of China′s Mainland. 4) The same range of length of side of square grid and dots of data collection can ensure the comparison of fractal dimensions of the mountains of China′s Mainland. The research is helpful to get the more understanding of fractal and fractal dimension, and ensure that the fractal studies would be scientific.

KEY WORDS: fractal; fractal dimension; series of square grids; China′s Mainland

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The fractal theory has been applied to a wide variety of fields after it was produced. The use of fractal analysis to geographical research has increased (MANDELBROT, 1967; XU *et al.*, 1993; NINA and DE, 1993; ANDRLE, 1996; GAO and XIA, 1996; ZHU and YANG, 2000; WILSON, 2001; CHRISTENSEN *et al.*, 2002), but large number of geographical phenomena still need to be studied, and many problems still remain unsolved. Although fractal dimensions of lots of phenomena were calculated by the box-counting method, the quantitative influence of series of square grids on them is ignored all the while. The issue is systematically discussed as a case study of mountains of China′s Mainland in this paper. It will undoubtedly help to get the more understanding of fractal and fractal dimension.

1 METHOD AND MATERIALS

1.1 Method

To study the quantitative influence of series of square grids on fractal dimensions, the spatial fractal character of the mountains of China′s Mainland is discussed in this paper, and its fractal dimensions are calculated by the box-counting method (GRASSBERGER, 1983).

To calculate the fractal dimension of an object, the object is covered with the square grids with variable lengths. Suppose the length of side of square grid is r, and the number of grids covering the object is N, according to the fractal theory,

$$N \propto r^{-D} \tag{1}$$

The lengths of the sides of the square grids are r_1, r_2, r_3, $\cdots$, r_k, and the numbers of grids covering the ob-

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ject are $N(r_1)$, $N(r_2)$, $N(r_3)$, ⋯, $N(r_k)$ correspondingly. The equivalent equation is

$$\log(N)=D\log(r)+A \quad (2)$$

where A is a constant, and D is the fractal dimension of the object, which equals to the absolute value of slope.

Series of square grids are used to discuss the quantitative influence of the mountains of China's Mainland on fractal dimensions by this method.

In addition, GIS software programs are used in the grid analysis, and the extracted data are analyzed using Excel 2000 software. It is self-evident that efficiency and data precision of the study are higher than those of manual operation. The technological flow for fractal analysis of mountains of China's Mainland is shown in Fig. 1.

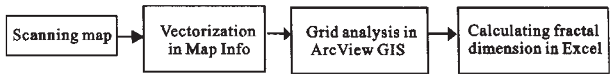

Fig.1 Flow chart of technology for fractal analysis

1.2 Data Sources

The data of the mountains of China's Mainland were from the *Atlas of Landsat Imagery of Main Active Faults in China* (Institute of Seismology, State Seismological Bureau, 1989). Sketch map of the mountains of China's Mainland is shown in Fig. 2.

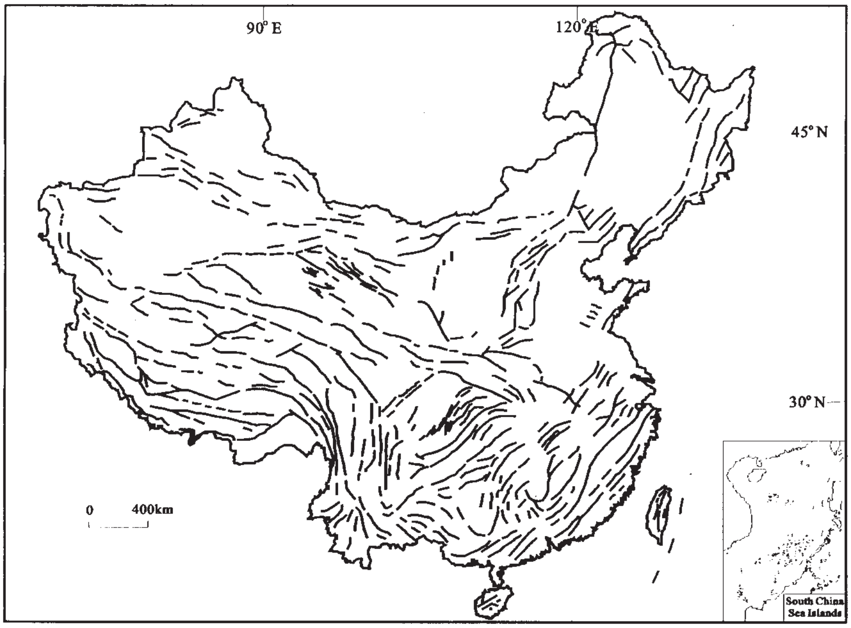

Fig. 2 Sketch map of the mountains of China's Mainland

2 INFLUENCE OF SERIES OF SQUARE GRIDS IN DIFFERENT RANGES ON FRACTAL DIMENSION

With the aid of GIS, the numbers of series of square grids are obtained. The results are listed in Table 1.

As shown in Table 1, when the lengths of sides of square grids of Pattern 1 are 500m and 499 500m, the numbers of grids covering the mountains of China's Mainland are 166 038 and 54 correspondingly. It is evident that the ranges of lengths of sides of Pattern 1–Pattern 15 in Table 1 are different.

By use of the data listed in Table 1, the diagrams showing the correlation between log(N) and log(r) are obtained. In view of limited space, only diagrams of Pattern 1 and Pattern 2 are listed (Fig. 3).

By linear regression analysis, equations (3) and (4) can be established as

$$\log(N) = -1.1319\log(r)+8.3959 \quad (3)$$
$$R = 0.9932$$
$$\log(N) = -1.1375\log(r)+8.4222 \quad (4)$$
$$R = 0.9932$$

where r is the length of side of square grid, N is the number of grids covering the mountains of China's Mainland correspondingly, and R is the correlation coefficient. The fractal dimension of the mountains of China's Mainland equals to the absolute value of the slope of various equations.

Based on the same data processing method, the fractal dimensions of the mountains of China's Mainland

Table 1 Numbers of square grids covering the mountains of China′s Mainland

Pattern 1		Pattern 2		Pattern 3		Pattern 4		Pattern 5	
r(m)	*N*	*r*(m)	*N*	*r*(m)	*N*	*r*(m)	*N*	*r*(m)	*N*
500	166038	600	138436	700	118708	800	103940	900	92405
4500	18743	4600	18407	4700	18003	4800	17629	4900	17260
9500	9046	9600	8992	9700	8903	9800	8802	9900	8707
49500	1741	49600	1731	49700	1719	49800	1715	49900	1711
99500	692	99600	686	99700	689	99800	687	99900	691
499500	54	499600	54	499700	54	499800	54	499900	54

Pattern 6		Pattern 7		Pattern 8		Pattern 9		Pattern 10	
r(m)	*N*	*r*(m)	*N*	*r*(m)	*N*	*r*(m)	*N*	*r*(m)	*N*
1000	83190	1100	75681	1200	69397	1300	64095	2000	41780
5000	16912	5100	16586	5200	16291	5300	16000	6000	14149
10000	8636	10100	8549	10200	8458	10300	8387	11000	7894
50000	1726	50100	1748	50200	1729	50300	1722	51000	1684
100000	692	100100	693	100200	699	100300	700	101000	676
500000	54	500100	54	500200	54	500300	54	501000	53

Pattern 11		Pattern 12		Pattern 13		Pattern 14		Pattern 15	
r(m)	*N*	*r*(m)	*N*	*r*(m)	*N*	*r*(m)	*N*	*r*(m)	*N*
3000	27970	4000	21070	5000	16912	6000	14149	7000	12171
7000	12171	8000	10718	9000	9564	10000	8636	11000	7894
12000	7234	13000	6721	14000	6238	15000	5867	16000	5508
52000	1644	53000	1609	54000	1571	55000	1524	56000	1512
102000	668	103000	665	104000	649	105000	636	106000	631
502000	53	503000	53	504000	53	505000	53	506000	52

Notes: *r* stands for length of side; *N*, number of square grids.

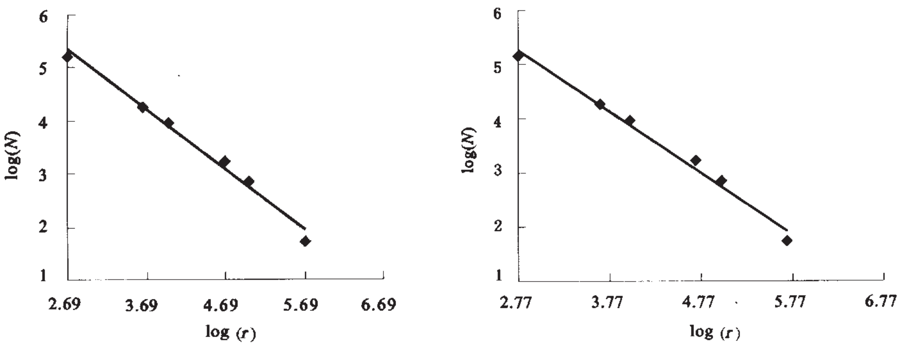

Fig. 3 Diagrams showing the correlation between log(N) and log(r) of Patterns 1 and Pattern 2

are obtained in different ranges of lengths of sides. The average lengths of sides of Pattern 1–Pattern 15 and corresponding fractal dimensions are listed in Table 2.

Their correlation coefficients are larger than the critical value of R test ($R_{0.05}$ = 0.8114), there are good correlations between log(N) and log(r) of Pattern1–Pattern 15.

From Table 2, it can be seen that the correlation coefficients are all more than 0.99, then the fractal character objectively exists in the mountains of China′s Mainland, it does not vary with the change of series of square grids covered in different ranges of lengths of sides. It also can be seen that the fractal dimensions of the mountains of China′s Mainland vary with the av-

Table 2 Average lengths of sides and fractal dimensions of patterns 1–15

Pattern	Average length of side (m)	Fractal dimension	Correlation coefficient
Pattern 1	110500	1.1319	0.9932
Pattern 2	110600	1.1375	0.9932
Pattern 3	110700	1.1416	0.9932
Pattern 4	110800	1.1456	0.9932
Pattern 5	100900	1.1486	0.9930
Pattern 6	111000	1.1513	0.9928
Pattern 7	111100	1.1538	0.9926
Pattern 8	111200	1.1563	0.9926
Pattern 9	111300	1.1590	0.9925
Pattern 10	112000	1.1795	0.9926
Pattern 11	113000	1.1974	0.9926
Pattern 12	114000	1.2121	0.9926
Pattern 13	115000	1.2258	0.9929
Pattern 14	116000	1.2388	0.9932
Pattern 15	117000	1.2528	0.9929

erage lengths of sides of series of square grids. By use of the data in Table 2, diagram of correlation between the average lengths of sides and corresponding fractal dimensions is shown as Fig. 4.

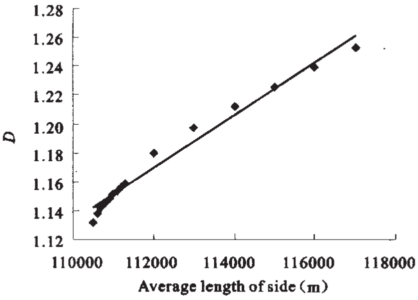

Fig. 4 Diagram of correlation between the average lengths of sides and fractal dimensions (D)

The linear relationship between the average lengths of sides and fractal dimensions is derived by linear regression analysis, it is

$$D = 2E(-05)r - 0.8797 \qquad (5)$$
$$R = 0.989$$

where r is the average length of side of square grid, D is corresponding fractal dimension of the mountains of China's Mainland, and R is the correlation coefficient.

The correlation coefficient is larger than the critical value of R test ($R_{0.05} = 0.5139$), there is a good correlation between the average length of side r and fractal dimension D.

3 INFLUENCE OF SERIES OF SQUARE GRIDS IN SAME RANGE ON FRACTAL DIMENSIONS

Based on GIS, the numbers of square grids in the same ranges are obtained. The results are listed in Table 3. Their ranges of lengths of sides of Pattern 16–Pattern 21 and Pattern 6 are the same(1000m to 500 000m).

Table 3 Number of square grids covering mountains of China's Mainland

Pattern 16		Pattern 17		Pattern 18		Pattern 19		Pattern 20		Pattern 21	
r(m)	N	r(m)	N	r(m)	N	r(m)	N	r(m)	N	r(m)	N
1000	83190	1000	83190	1000	83190	1000	83190	1000	83190	1000	83190
2000	41780	2500	33498	3000	27970	3500	24039	4000	21070	4500	18743
7000	12171	7500	11403	8000	10718	8500	10088	9000	9564	9500	9046
47000	1849	47500	1833	48000	1786	48500	1784	49000	1784	49500	1741
97000	729	97500	717	98000	711	98500	703	99000	701	99500	692
500000	54	500000	54	500000	54	500000	54	500000	54	500000	54

Notes: r stands for length of side; N, number of square grids.

The diagrams showing the correlation between log(N) versus log(r) of Pattern 16 and Pattern 17 is established by the data of Pattern 16 and Pattern 17 listed in Table 3 (Fig. 5).

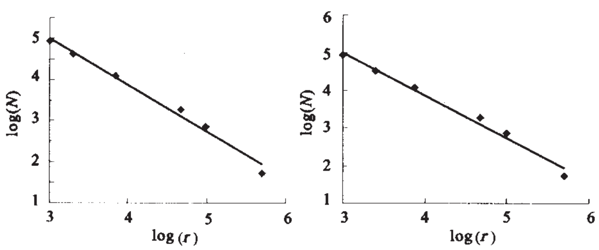

Fig. 5 Diagrams showing the correlation between log(N) and log(r) of Pattern 16 and Pattern 17

By linear regression analysis, equations (6) and (7) can be established as

$$\log(N) = -1.1391 \log(r) + 8.4234 \qquad (6)$$
$$R = 0.9935$$
$$\log(N) = -1.1428 \log(r) + 8.4421 \qquad (7)$$
$$R = 0.9934$$

where r is the length of side of square grid, N is the number of grids covering the corresponding mountains of China's Mainland, and R is the correlation coefficient. The fractal dimension of the mountains of China's Mainland equals to the absolute value of the slope of the equation.

Table 4 shows the average lengths of sides of patterns 16–21, Pattern 6 and corresponding fractal dimensions of the mountains of China's Mainland.

Table 4 Average lengths of sides and fractal dimensions of patterns 16–21 and Pattern 6

Pattern	Average length of side (m)	Fractal dimension	Correlation coefficient
Pattern 16	109000.0	1.1391	0.9935
Pattern 17	109333.3	1.1428	0.9934
Pattern 18	109666.7	1.1459	0.9934
Pattern 19	110000.0	1.1477	0.9933
Pattern 20	110333.3	1.1487	0.9930
Pattern 21	110666.7	1.1506	0.9930
Pattern 6	111000.0	1.1513	0.9928

From Table 4, it can also be seen that the fractal character objectively exists in the mountains of China's Mainland, it does not vary with the change of series of square grids. In addition, the fractal dimensions of the mountains of China's Mainland also vary with the average lengths of sides of series of square grids. The diagram of correlation between the average lengths of sides and fractal dimensions can be established by use of the data in Table 4 (Fig. 6).

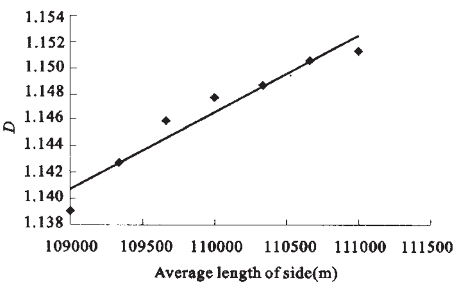

Fig. 6 Diagram of correlation between the average lengths of sides and fractal dimensions

By linear regression analysis, the linear relationship between the average lengths of sides and fractal dimensions is expressed as

$$D = 6E(-06)r + 0.4984 \quad (8)$$
$$R = 0.9697$$

where r is the average length of side of square grid, D is fractal dimension of the corresponding mountains of China's Mainland, and R is the correlation coefficient.

The correlation coefficient is larger than the critical value of R test ($R_{0.05} = 0.7545$), so there is a good correlation between the average length of sides r and fractal dimension D too.

4 INFLUENCE OF DIFFERENT DOTS OF DATA COLLECTION ON FRACTAL DIMENSIONS

At the aid of GIS, the numbers of square grids for different dots are obtained. The results are listed in Table 5 and Table 6. Their ranges of lengths of sides are from 1000m to 500 000m, but the dots of data collection in Table 5 and Table 6 are different. There are 6 dots in Table 5, and 23 dots in Table 6.

Fig.7 is established by the data listed in Table 5 and Table 6.

By linear regression analysis, equations (9) and (10) can be established as

$$\log(N) = -1.1513\log(r) + 8.4901 \quad (9)$$
$$R = 0.9928$$
$$\log(N) = -1.148\log(r) + 8.5041 \quad (10)$$
$$R = 0.9927$$

where r is the length of side of square grid, N is the corresponding number of grids covering the mountains of China's Mainland, and R is the correlation coefficient. The fractal dimensions of the mountains of China's Mainland are 1.1513 and 1.1480 based on Table 5 and Table 6 respectively. Thus, different dots of data collection can affect the fractal dimension of the mountains of China's Mainland.

Table 5 Numbers of square grids for 6 dots of data collection

r(m)	1000	5000	10000	50000	100000	500000
N	83190	16912	8636	1726	692	54

Notes: r stands for length of side; N, number of square grids.

Table 6 Numbers of square grids covering mountains of China's Mainland for 23 dots of data collection

r (m)	N	r(m)	N	r(m)	N
1000	83190	9000	9564	80000	948
2000	41780	10000	8636	90000	806
3000	27970	20000	4436	100000	692
4000	21070	30000	3022	200000	236
5000	16912	40000	2232	300000	117
6000	14149	50000	1726	400000	78
7000	12171	60000	1385	500000	54
8000	10718	70000	113		

Notes: r stands for length of side; N, number of square grids.

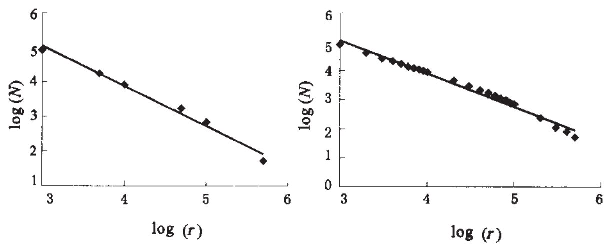

Fig. 7 Diagrams showing the correlation between log (N) and log (r) of Table 5 and 6

5 CONCLUSIONS

According to the above analysis, some significant conclusions are drawn:

(1) Although the fractal character objectively exists in the mountains of China's Mainland, and it does not vary with the changes of series of square grids, the fractal dimensions of the mountains of China's Mainland are different with these changes.

(2) The fractal dimensions of the mountains of China's Mainland vary with the average lengths of sides of series of square grids. The fractal dimension of the mountains of China's Mainland is the function of the average length of side of square grid. They conform to the formula $D=f(r)$ (where D is the fractal dimension, and r is the average length of side of square grid).

(3) Different dots of data collection can affect the fractal dimension of mountains of China′s Mainland.

(4) The same range of length of side of square grid and dots of data collection can ensure the comparison of fractal dimensions of the mountains of China′s Mainland. It is just the same to the other fractal studies and undoubtedly helps to get the more understanding of fractal and fractal dimension.

In a word, quantitative analysis was made to the influence of series of square grids on fractal dimensions, and it is important to ensure that the fractal studies would be scientific.

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