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THE APPLICATION OF FRACTAL STUDY IN FORECASTING FLOOD CALAMITIES IN WUZHOU, GUANGXI

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ABSTRACT: As a kind of sudden and common natural disaster, flood is characterized by the temporal fraction. A date series of floods was established according to the catastrophic records in Wuzhou, Zhuang Autonomous Region of Gaungxi, P. R. China since 1949. By the means of the model of fractional brownian motion, we can imitate the temporal sequence of past and forecast the developing trend in future, on the basis of which, the R/S analysis was employed to calculate the temporal sequence, the 'H' exponent and function $R(\tau)/S(\tau)$ were determined. It is anticipated that the next catastrophic flood in Wuzhou will happen in 1999.

KEY WORDS: fractal theory, fractional brownian motion model, '*H*' exponent, flood calamity

1　FRACTAL THEORY AND FLOOD CALAMITIES

The most regular geometric objects which could be measured by lengths of specificity are the issue of classical Euclidean geometry. However, there are numerous irregular geometric figures and structures of complexity in the nature, which are the targets of fractal geometry, widely adopted in various social and natural fields since its foundation in the 1970s by an American scholar, B. B. Mandelbrot.

The fractal theory is mostly concerned with the self-similarity or self-affinity in a system. The former refers to the partial similarity to the entirety, i. e. the repeat appearance of structure featured in different scales. The pattern of similarity of constitutional part with its entirety is called fraction. The solid base of self-similarity defined by the fractal theory was founded by the wide practical demands such as Hicle Repeating Law in biology, embedding structure of natural events in different levels etc. Fraction is quantitatively described by fractal dimension, FD for short. FD is the exponent of the objective size to the measurement scale, frequently defined by a power function indicating the irregularity, complexity and confusion by quantities. Taking r as the scale of size measurement and $N(r)$ as the result of survey for a given system, when the varying r and $N(r)$ fit in the relationship with decreasing r:

$$N(r) = Cr^{-D} \propto r^{-D}$$

then the system is treated as one with fractal structure, where D is the dimensions and C a constant.

Dramatic damages are caused by the flood calamities. The statistics showed that the area suffered from that disaster during the period of 1951–1990 in China was up to 8 425 000 ha/a, about 40% of the total loss resulted from various disasters (Shi, 1996). As examining the flood process a discontinuous feature in time was discovered. And the process also associated with the characteristics of mutual mosaic in varying time spanning. For instance, the over-raining decades of the 1930 s and the 1950 s met the most frequent floods, up to 8 and 11 respectively in the total 34 bursts from 1915 to 1985. While mere 4 floods endeavored in the 20 years from 1960 to 1980 when rain was less. Entering the 1980s, the suffering area of floods showed an increasing trend in relation to that in the 1950s though the rain was distinctly less (Shi, 1996). Once again the floods have become frequent since 1990, as records of 1991 in Taihu Lake and the Huaihe River drainage; 1994 mainly in southern part of China; 1995 in Hunan, Jiangxi and eastern Liaoning provinces; 1996 in Hunan, Hubei, Hebei, Henan and other places.

The history does not show repeated floods in a certain period, and also a definite interval of a given flood of magnitude. Rather, it exhibits a distinct alternation of peaks and troughs in terms of identified bursts. This means that a relative calm stage follows the active phase, and then another comes. It is very often to have 2 or 3 flood years continuously during an active cycle. In one word, the flood calamities bring neither definitely nor randomly about, but happen with high probability of periodicity, or chaos. On the view of fractal theory, the floods show similar structure in the time dimension, i.e., having a kind of similarity. Actually, it is a typical irregular Cantor Set.

2 THE ANALYSIS OF THE FLOOD CALAMITIES IN WUZHOU CITY

Wuzhou, a city in the southern part of Zhuang Autonomous Region of Guangxi, is located at the juncture of the Xijiang, Guijiang and Xunjiang rivers. With the mountainous higher portion in the north, the southern Wuzhou displays a typical hilly land of low elevation. The climate of Wuzhou is marine type subject to the occasional typhoon and monsoon prevailing in the tropical area of southern Asia. The mean temperature in Wuzhou is 21.1℃ annually, 11.8℃ in the coldest month (January), and 28.4℃ in the warmest month (July). Raining season of Wuzhou is from April to September, sharing up to 80% of the annual precipitation of 1506.9 mm. In these months, torrential rains make up 89% of the frequency in a year, with the heaviest all in the period from April to August. The areal distribution of the rain is increasing from northeast, 1600–1700 mm/a, to southwest, 1300–1500 mm/a. The extremes are 1925.9 mm/a, highest in 1962, and 978.4 mm/a, lowest in 1956. The heaviest daily rainfall is recorded on the June 12, 1966, up to 295.4 mm. In addition, 90% of runoff in Guangxi discharges to Guangdong through the Xunjiang River. Wuzhou, at its lower reach, thus suffers easily from frequent floods.

The history has more than 110 flood calamities in 974–1955 A.D. The alarming water level for Wuzhou is 15 m. When it is over 18 m, the streets would be inundated. The peak average level of floods was 20.18 m in the period of 1900–1995, the years with level height more than 23 m was 18 a, making up 19% of the total; 20–22.29 m 32a, 33.7%; 18–19.99 m 22a, 23%. The catastrophic floods in this century arose in 1902, 1915, 1918, 1924, 1944, 1949, 1954, 1957, 1962, 1966, 1968, 1970, 1974, 1976, 1978, 1682, 1984, 1988, 1994. One of them in 1966, had the daily rainfall of 295.4 mm and the rain strength up to 74.2 mm/h on June 12, causing the inundated paddy field of 666.67 ha, damaged houses of 41 and broken highway in 75 sites. During July–August of the same year, the flood with 24.5 m level in the Xunjiang River and the Guijiang River resulted in inundated paddy field of 3478 ha and destroyed houses of 145.

3 THE METHOD FOR FORECASTING FLOOD CALAMITIES

One of the successful examples to apply the fractal theory was given by Zhou (1996), in which the historical flood calamities of the Huaihe River were analyzed and the temporal dimension was calculated. We are to employ a model, fractional brownian motion introduced firstly by B. B. Mandelbrot (Mandelbrot *et al.*, 1968; Feder, 1988; Huang, 1990), to imitate the temporal sequence of the Wuzhou floods in the past and to anticipate the developing trend in the future.

3.1 Fractional Brownian Motion Model

The principle of the fractionl brownian motion model could be described as follows:

Supposing a particle is moving randomly along the x-axis with the pace length of x in a given time interval of τ, then the distribution density function of x is:

$$f(x, \tau) = \frac{1}{\sqrt{2\pi}\sqrt{2D\tau}}\exp\left[-\frac{x^2}{2(\sqrt{2D\tau})^2}\right] = \frac{1}{\sqrt{4\pi D\tau}}\exp\left(-\frac{x^2}{4D\tau}\right)$$

where D is the diffusion coefficient; τ time interval. If $x_1, x_2, \ldots, x_n$ are the pace sequence of time as a independent variable, after n paces the particle should locate at a point of x-axis:

$$B(t = n\tau) = \sum_{i=1}^{n} x_i$$

The $B(t)$ related to the time, obtained in this way, is called the brownian function. For a given time t_0, the scattering density of $B(t) - B(t_0)$ is:

$$f(B(t) - B(t_0)) = \frac{1}{\sqrt{(4\pi D|t - t_0|}} \exp\left(-\frac{[B(t) - B(t_0)^2]}{4D|t - t_0|}\right)$$

Taking

$$\xi = \frac{B(t) - B(t_0)}{\sqrt{2D|t - t_0|}} = \frac{B(t) - B(t_0)}{\sqrt{2D\tau}(|t - t_0|/\tau)^{1/2}}$$

Then we get a model, brownian motion model, following the normal distribution.

If
$$\xi = \frac{B(t) - B(t_0)}{\sqrt{2D\tau}(|t - t_0|/\tau)^H}$$

$$\left(H \neq \frac{1}{2}, 0 < H < 1\right)$$

The model is called fractional brownian motion model, and H is Hurst exponent. For the brownian function $B_H(t)$ in the model:

$$E(B_H(t) - B_H(t_0)) = 0$$

$$V(t - t_0) = E[B_H(t) - B_H(t_0)]^2 = 2D\tau(|t - t_0|/\tau)^{2H}$$

Correlation function is:

$$r(t) = \frac{E\{[B_H(0) - B_H(-t)][B_H(t) - B_H(0)]\}}{\sqrt{E[B_H(0) - B_H(-t)]^2} \cdot \sqrt{E[B_H(t) - B_H(0)]^2}}$$

Taking $B_H(0) = 0$, $\tau = 1$, $2D\tau = 1$, then:

$$r(t) = E\{-B_H(-t) \cdot B_H(t)\}/E[B_H(t)]^2 = 2^{2H-1} - 1$$

It can be seen $\gamma(t) = 0$ when $H = 1/2$, corresponding to the general brownian motion; and to fractal brownian motion when $H \neq 1/2$, in this case $\gamma(t) \neq 0$ and $\gamma(t)$ is nothing to de with t, showing the long distance correlation. If $H < 1/2$, when $\gamma(t) < 0$, meaning a negative correlation between prior and future increments. i.e. exhibiting an antipersistence. Vise versa, indicating a positive correlation. Hence, the value of H determines the trend of fractional brownian motion.

3.2 The Calculation of 'H' exponent

The 'H' exponent was first brought into use by H.E. Hurst in 1965. The statistical method, R/S analysis, adopted by him has found wide application in the fractal theory. Numerous natural phenomena such as runoff of rivers and mud deposition, as found by Hurst, have showed $H > 1/2$ (Hurst *et al.*, 1965). When brownian motion was firstly introduced by B.B. Mandelbrot, the domain of H was extended to $0 < H < 1$, and the results (Mandelbrot *et al.*, 1968; Feder, 1988) gained were:

$$R(\tau)/S(\tau) = (a\tau)^H$$

where $R(\tau)$ is the extreme deviation; $S(\tau)$ the standard deviation and a constant.

$$R(\tau) = \max_{1 \leqslant i \leqslant \tau} x(i, \tau) - \min_{1 \leqslant i \leqslant \tau} x(i, \tau)$$

$$\bar{x}(\tau) = \frac{1}{\tau}\sum_{i=1}^{\tau} x(i, \tau)$$

$$S(\tau) = \left[\frac{1}{\tau}\sum_{i=1}^{\tau}(x(i, \tau) - \bar{x}(i, \tau))^2\right]^{1/2}$$

From equation $R(\tau)/S(\tau) = (a\tau)^H$, we can also get:

$$\ln[R(\tau)/S(\tau)] = H\ln a + H\ln\tau$$

the $R(\tau)/S(\tau)$ ($\tau = 2, 3, \ldots$) could be calculated by the original data $\{x_i\}$, Making the linear regression of $\ln[R(\tau)/S(\tau)]$ to $\ln\tau$, then the slope of the line is the estimation of ' H ' exponent.

There is close relation between ' H ' exponent and the dimension of fractional brownian motion, which founds the base of the Mandelbrot fractal geometry (Feder, 1988; Huang, 1990). It describes the irregularity or chaos of relation of τ to $R(\tau)/S(\tau)$, indicating the trend of fractional brownian motion defined by ' H ' exponent.

The method mentioned above is so called R/S analysis, which shows wide application in practice. So, it is decided to anticipate the flood calamities for Wuzhou by it.

4 FORECASTING EXAMPLE

It is defined in Z exponent, showing the strength of drought and flood, that flood extreme happens when $Z > 1.645$, and heavy flood with $1.037 < Z \leqslant 1.645$. Putting those two into the category of catastrophic flood (CF), then the years suffered from the CF for Wuzhou is 1949, 1954, 1957, 1962, 1966, 1968, 1970, 1974, 1976, 1978, 1982, 1984, 1988, 1994 etc. If take 1948 as the beginning year of the sequence, the $x(i, \tau)$ displays the order:

$x(i, \tau)\{1, 6, 9, 14, 18, 20, 22, 26, 28, 30, 34, 36, 40, 46\}$ ' H ' exponent

According to the mentioned method to find ' H ' exponent, the following order for $R(\tau)$ and $S(\tau)$ could be determined:

$\tau = 2$	$R(2) = 5$	$S(2) = 2.5$	$R(2)/S(2) = 2.0000$
$\tau = 3$	$R(3) = 8$	$S(3) = 3.30$	$R(3)/S(3) = 2.4242$
$\tau = 4$	$R(4) = 13$	$S(4) = 4.7170$	$R(4)/S(4) = 2.7560$
$\tau = 5$	$R(5) = 17$	$S(5) = 5.9531$	$R(5)/S(5) = 2.8557$
$\tau = 6$	$R(6) = 19$	$S(6) = 6.6750$	$R(6)/S(6) = 2.8464$
$\tau = 7$	$R(7) = 21$	$S(7) = 7.2196$	$R(7)/S(7) = 2.9087$
$\tau = 8$	$R(8) = 25$	$S(8) = 8.0312$	$R(8)/S(8) = 3.1129$
$\tau = 9$	$R(9) = 27$	$S(9) = 8.6795$	$R(9)/S(9) = 3.1108$
$\tau = 10$	$R(10) = 29$	$S(10) = 9.2434$	$R(10)/S(10) = 3.1374$
$\tau = 11$	$R(11) = 33$	$S(11) = 10.0223$	$R(11)/S(11) = 3.2927$
$\tau = 12$	$R(12) = 35$	$S(12) = 10.6953$	$R(12)/S(12) = 3.2725$
$\tau = 13$	$R(13) = 39$	$S(13) = 11.5349$	$R(13)/S(13) = 3.8180$
$\tau = 14$	$R(14) = 45$	$S(14) = 12.7375$	$R(14)/S(14) = 3.5329$

Linear regression $\ln[R(\tau)/S(\tau)]$ about $\ln\tau$ contributes the ' H ' value through least square method:

$$H = 0.2474$$

And: $\gamma(t) = 2^{2H-1} - 1 = -0.2954 < 0$

Its zygote dimension is:

$$D_B = 2 - H = 1.7526 > 1.5$$

Indicating the antipersistence in fractional brownian motion. Also it is easy to find:

$$R(\tau)/S(\tau) = (11.3045\tau)^{0.2474}$$

To have high precision, the segmentation imitation could be adopted. When $\tau < 9$, and the period from 1949 to 1976, then:

$$H = 0.2893$$

$$D_B = 1.7107$$

$$\gamma(t) = -0.2533 < 0$$

$$R(\tau)/S(\tau) = (6.6042\tau)^{0.2893}$$

When $\tau \geqslant 9$, i, e., the imitation period from 1976 to 1995, we get:

$$H = 0.2732$$

$$D_B = 1.7268$$

$$\gamma(t) = -0.2700 < 0$$

$$R(\tau)/S(\tau) = (6.8346\tau)^{0.2732}$$

The previous study (Zhou, 1996) suggested a counter-correlation between the temporal dimension value and the flood calamity cycle, the less the dimension value is the longer the flood cycle is. Due to the close relation of 'H' exponent to the fractional brownian motion, the cycle length could be reflected in the 'H' exponent. The period of 1958–1976 saw frequent floods in Wuzhou, having shorter cycle length, with higher 'H' exponent, fitting well with the result of segmentation imitation. Since the late 1970s, the catastrophic floods showed longer cycle, then the H value has dropped down.

Consequently, the natural trend of the catastrophic floods for WuZhou City could be forecasted: when $\tau = 15$, by calculation of $R(\tau)/S(\tau)$, $X(15) = 51$. That means that the coming first catastrophic flood is probably in 1999.

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